Lecture 2

2018/2019

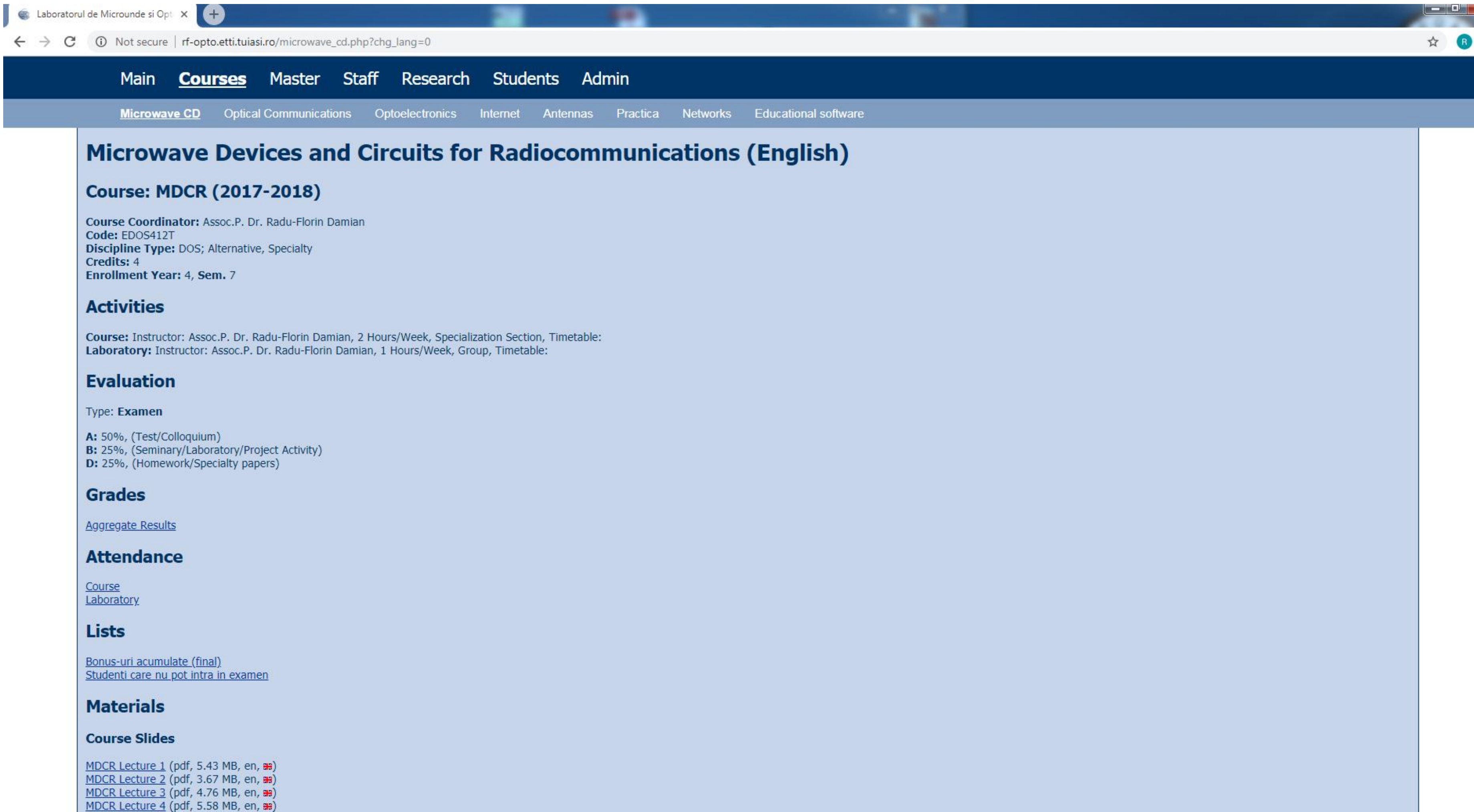
Microwave Devices and Circuits for Radiocommunications

2018/2019

- 2C/1L, **MDCR**
- **Attendance at minimum 7 sessions (course + laboratory)**
- Lectures- **associate professor Radu Damian**
  - Friday 09-11, ? III.34, II.13
  - E – 50% final grade
  - **problems** + (2p atten. lect.) + (3 tests) + (bonus activity)
    - 3p=+0.5p
  - **all materials/equipments authorized**
- Laboratory – **associate professor Radu Damian**
  - Wednesday 12-14, II.12 odd weeks
  - L – 25% final grade
  - P – 25% final grade

Materials

- http://rf-opto.etti.tuiasi.ro



Laboratorul de Microunde si Opt
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Main Courses Master Staff Research Students Admin

Microwave CD Optical Communications Optoelectronics Internet Antennas Practica Networks Educational software

Microwave Devices and Circuits for Radiocommunications (English)

Course: MDCR (2017-2018)

Course Coordinator: Assoc.P. Dr. Radu-Florin Damian
Code: EDOS412T
Discipline Type: DOS; Alternative, Specialty
Credits: 4
Enrollment Year: 4, **Sem.** 7

Activities

Course: Instructor: Assoc.P. Dr. Radu-Florin Damian, 2 Hours/Week, Specialization Section, Timetable:
Laboratory: Instructor: Assoc.P. Dr. Radu-Florin Damian, 1 Hours/Week, Group, Timetable:

Evaluation

Type: **Examen**

A: 50%, (Test/Colloquium)
B: 25%, (Seminary/Laboratory/Project Activity)
D: 25%, (Homework/Specialty papers)

Grades

Aggregate Results

Attendance

Course
Laboratory

Lists

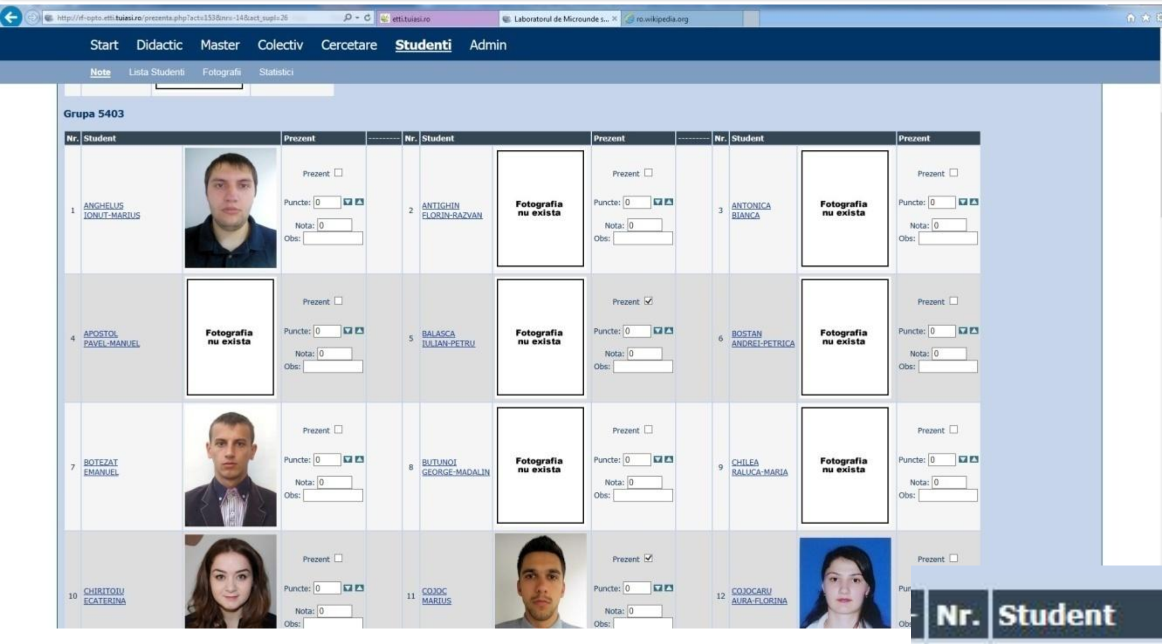
Bonus-uri acumulate (final)
Studenti care nu pot intra in examen

Materials

Course Slides

MDCR Lecture 1 (pdf, 5.43 MB, en,)
MDCR Lecture 2 (pdf, 3.67 MB, en,)
MDCR Lecture 3 (pdf, 4.76 MB, en,)
MDCR Lecture 4 (pdf, 5.58 MB, en,)

Photos



Start
Didactic
Master
Colectiv
Cercetare
Studenti
Admin
Note
Lista Studenti
Fotografii
Statistici
Grupa 5403
Nr.
Student
Prezent
1 ANGHELUS IONUT-MARIUS
2 ANTIGHIN FLORIN-RAZVAN
3 ANTONICA BIANCA
4 APOSTOL PAVEL-MANUEL
5 BALASCA IULIAN-PETRU
6 BOSTAN ANDREI-PETRICA
7 BOTEZAT EMANUEL
8 BUTUNOI GEORGE-MADALIN
9 CHILEA RALUCA-MARIA
10 CHIRITOIU ECATERINA
11 COJOC MARIUS
12 COJOCARU AURA-FLORINA
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Puncte:
Nota:
Obs:

Nr.
Student
Prezent
2
ANTIGHIN FLORIN-RAZVAN
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Nota: 0
Obs:

Access

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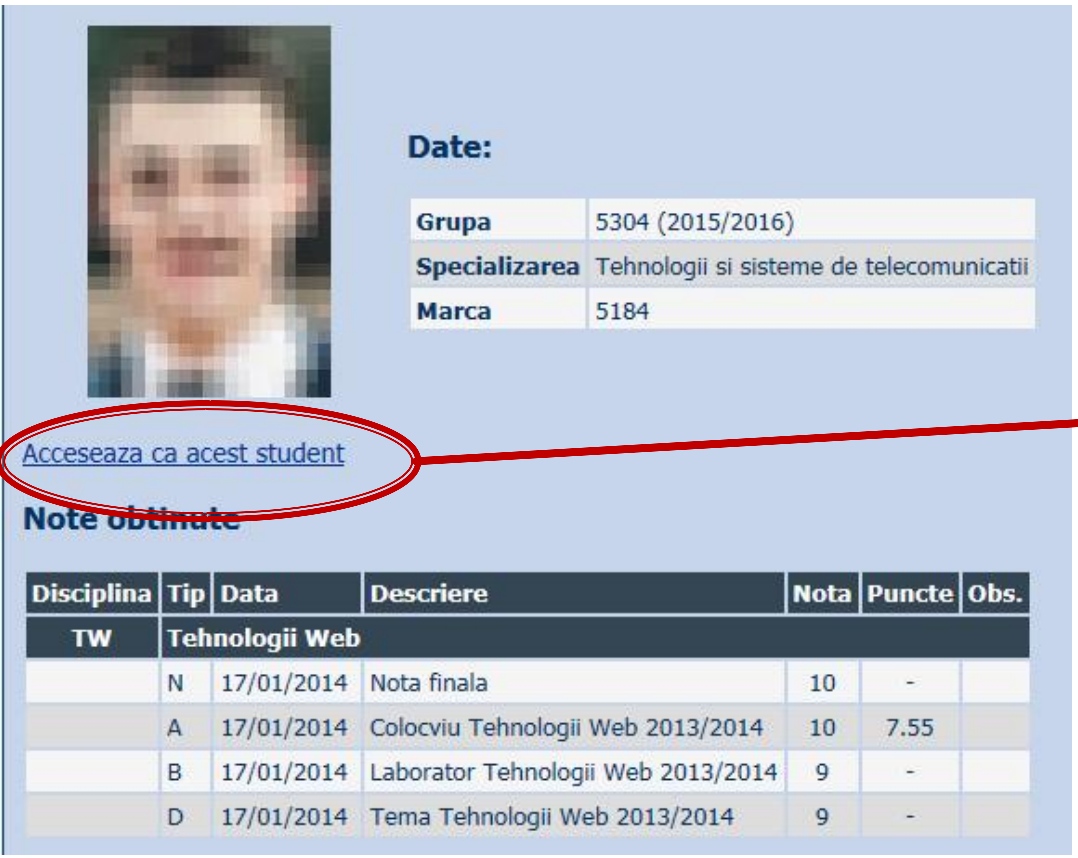


Date:

Grupa	5304 (2015/2016)
Specializarea	Tehnologii si sisteme de telecomunicatii
Marca	5184

Acceseaza ca acest student

Note obtinute

Disciplina	Tip	Data	Descriere	Nota	Puncte	Obs.
TW	Tehnologii Web					
	N	17/01/2014	Nota finala	10	-	
	A	17/01/2014	Colocviu Tehnologii Web 2013/2014	10	7.55	
	B	17/01/2014	Laborator Tehnologii Web 2013/2014	9	-	
	D	17/01/2014	Tema Tehnologii Web 2013/2014	9	-	

Nume

[redacted]

Email

Cod de verificare

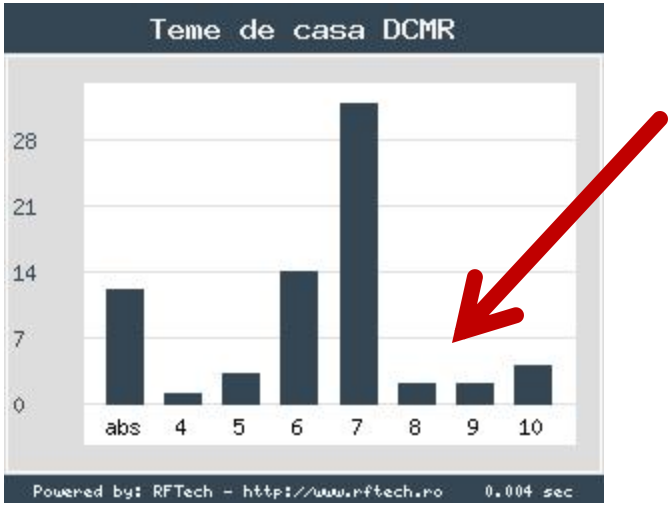
344bd9f

Trimite

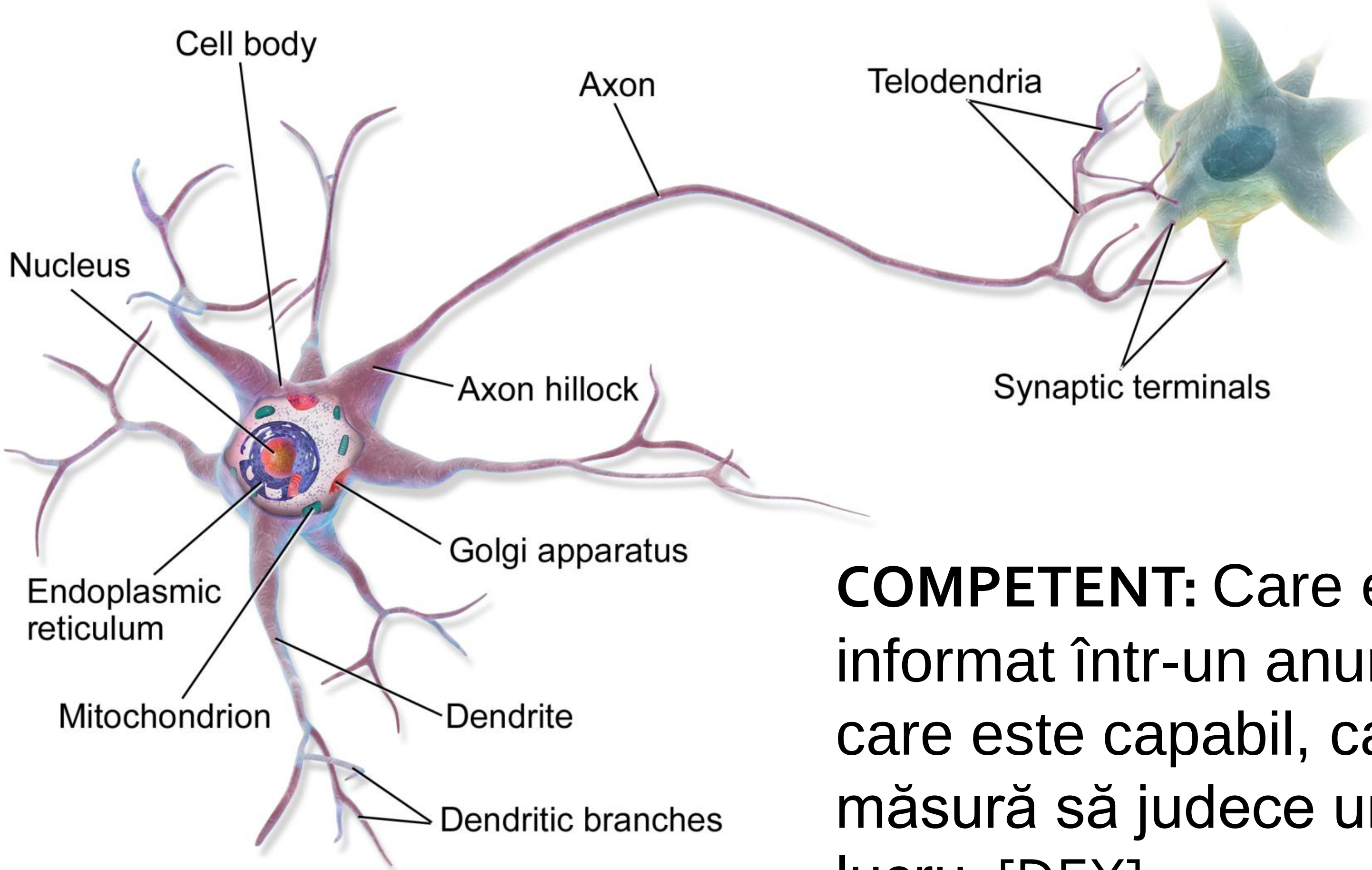
Proiect 2018/2019

- factorul “andrei” = -2p

2017/8



Course Objective

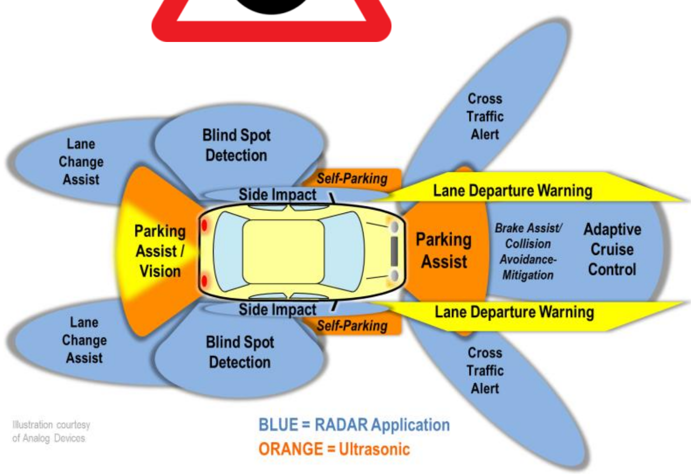


COMPETENT: Care este bine informat într-un anumit domeniu; care este capabil, care este în măsură să judece un anumit lucru. [DEX]

~1930

Technology

> 2010



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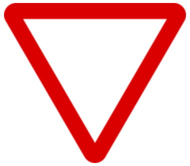


Examen: Logarithmic scales

dB $= 10 \bullet \log_{10} (P_2 / P_1)$

dBm $= 10 \bullet \log_{10} (P / 1\ mW)$

0 dB	= 1
+ 0.1 dB	= 1.023 (+2.3%)
+ 3 dB	= 2
+ 5 dB	= 3
+ 10 dB	= 10
-3 dB	= 0.5
-10 dB	= 0.1
-20 dB	= 0.01
-30 dB	= 0.001

0 dBm	= 1 mW
3 dBm	= 2 mW
5 dBm	= 3 mW
10 dBm	= 10 mW
20 dBm	= 100 mW
-3 dBm	= 0.5 mW
-10 dBm	= 100 μW
-30 dBm	= 1 μW
-60 dBm	= 1 nW

[dBm] + [dB] = [dBm]

[dBm/Hz] + [dB] = [dBm/Hz]

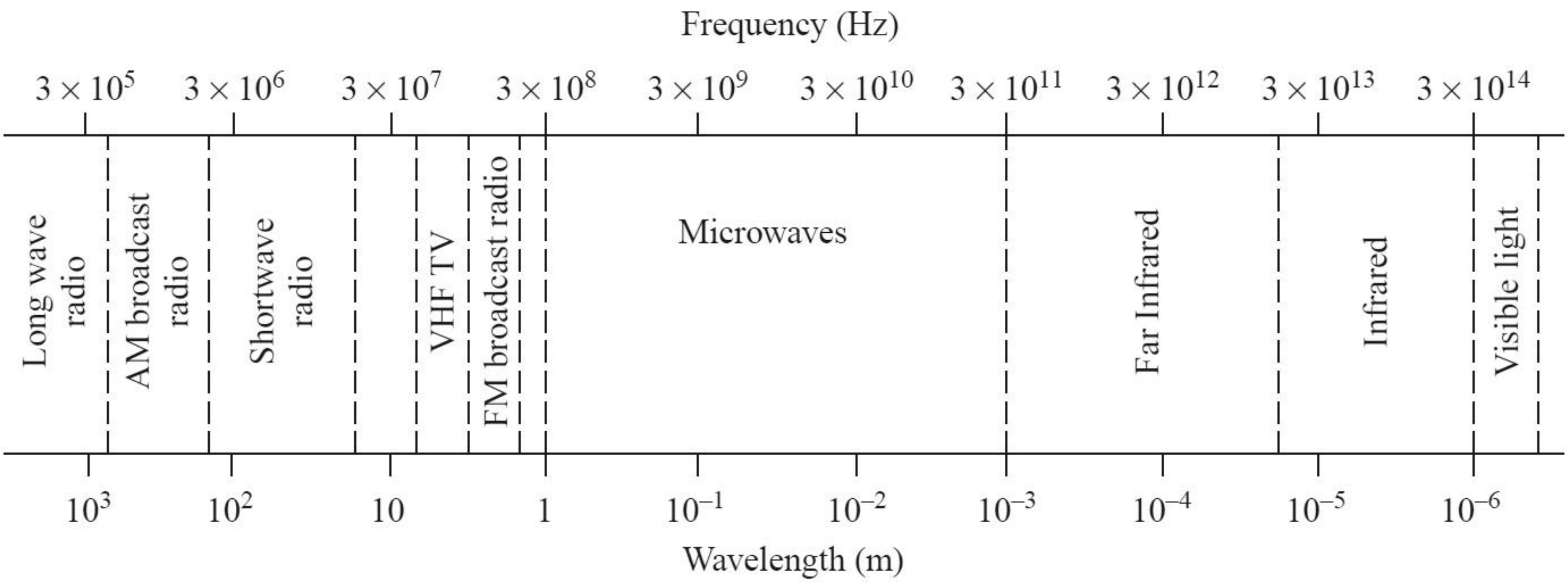
[x] + [dB] = [x]

Examen

- Complex numbers arithmetic**!!!!**
- $z = a + j \cdot b$; $\mathbf{j^2 = -1}$

Introduction

Microwaves



- typically
  - f ≈ 1÷3GHz – 300GHz
  - λ ≈ 1mm – 10cm

~ Microwaves

- Electrical Length (Phase Length)
  - l – physical length
  - E = β·l – electrical Length

$$E = \beta \cdot l = \frac{2\pi}{\lambda} \cdot l = 2\pi \cdot \left(\frac{l}{\lambda} \right)$$

$$E = \beta \cdot l = \frac{2\pi}{c_0} \cdot \left(l \cdot f \cdot \sqrt{\varepsilon_r} \right)$$

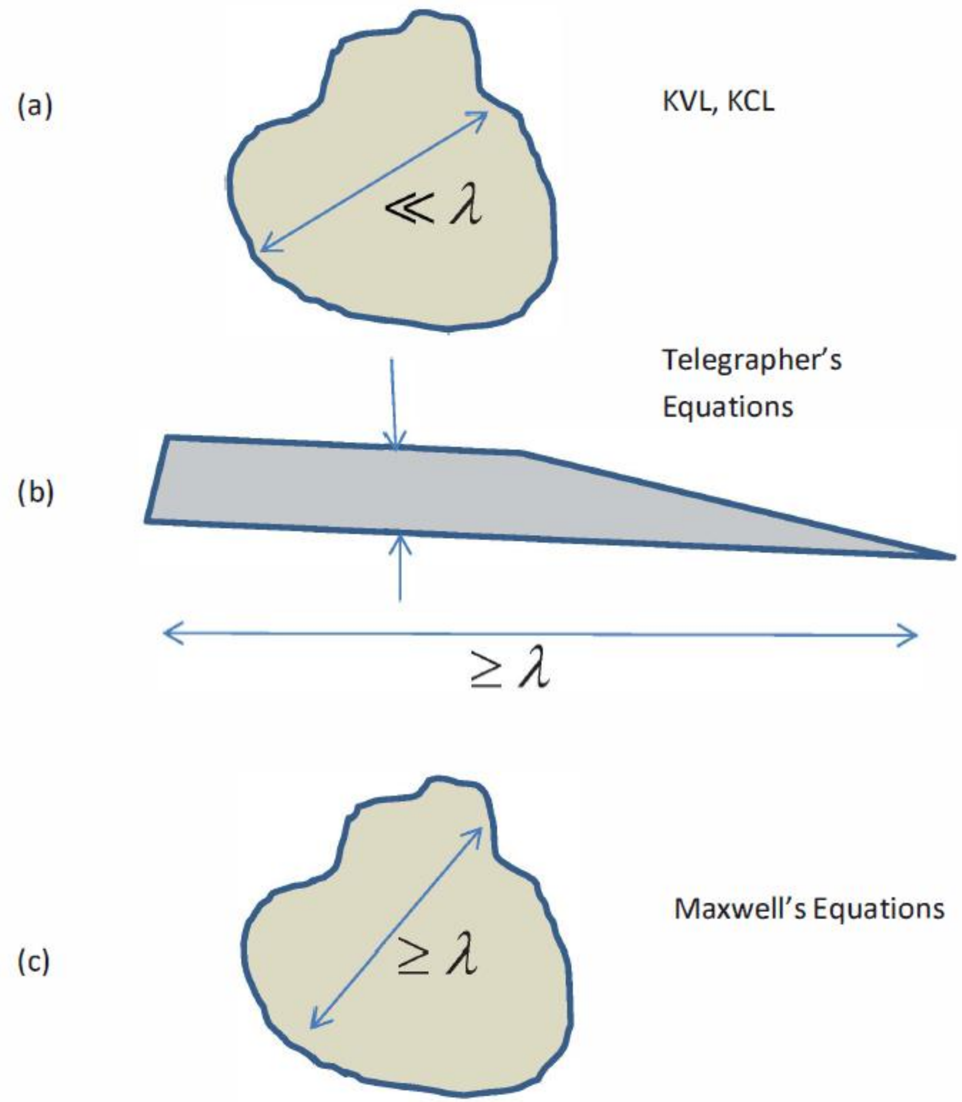
V, I vary
~ useless

- Dependency
  - antenna gain
  - Radar cross-section

Electrical Length

- Behavior (and description) of any circuit depends on his electrical length at the particular frequency of interest
  - E≈0 → Kirchhoff
  - E>0 → wave propagation

$$E = \beta \cdot l = \frac{2\pi}{\lambda} \cdot l = 2\pi \cdot \left(\frac{l}{\lambda} \right)$$



Maxwell's Equations

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \times H = \frac{\partial D}{\partial t} + J$$

$$\nabla \cdot D = \rho$$

$$\nabla \cdot B = 0$$

$$\nabla \cdot J = -\frac{\partial \rho}{\partial t}$$

- Constitutive equations

$$D = \varepsilon \cdot E$$

$$B = \mu \cdot H$$

$$J = \sigma \cdot E$$

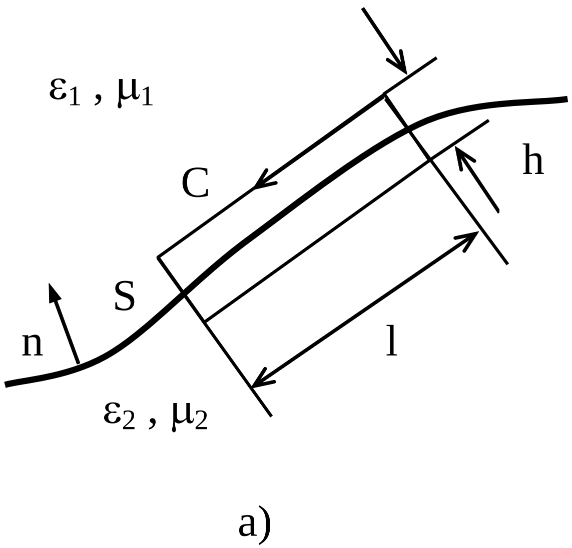
- Vacuum

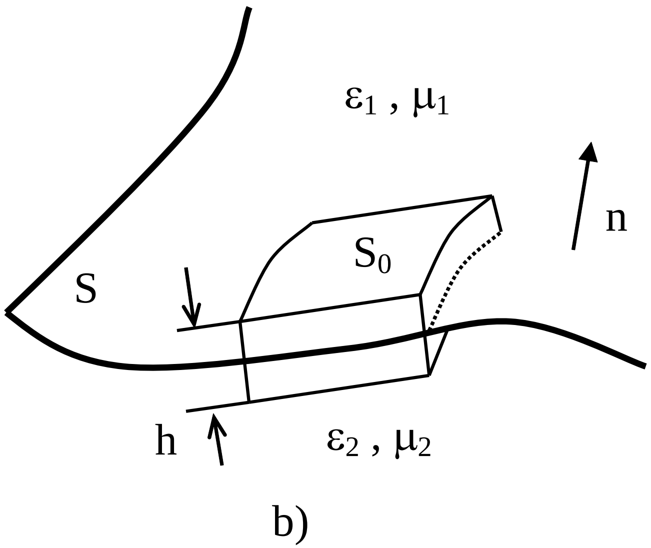
$$\mu_0 = 4\pi \times 10^{-7} \; H/m$$

$$\varepsilon_0 = 8{,}854 \times 10^{-12} \; F/m$$

$$c_0 = \frac{1}{\sqrt{\varepsilon_0 \cdot \mu_0}} = 2{,}99790 \cdot 10^8 \; m/s$$

Interface conditions on the interface between two different media





$$n \times (E_1 - E_2) = 0 \qquad n \cdot (D_1 - D_2) = \rho_S$$

$$n \times (H_1 - H_2) = J_S \qquad n \cdot (B_1 - B_2) = 0$$

- If one of the media is a perfect conductor (metal) all fields are annulled inside

Electromagnetic fields with harmonic time dependence

$$X = X_0 e^{j\cdot\omega\cdot t} \qquad \frac{\partial X}{\partial t} = j\cdot\omega\cdot X$$

$$g(\omega) = \int_{-\infty}^{\infty} f(t)\cdot e^{-j\omega t}\,dt \qquad f(t) = \int_{-\infty}^{\infty} g(\omega)\cdot e^{j\omega t}\,d\omega$$

- Maxwell's Equations more simple

$$\nabla^2 E + \omega^2 \varepsilon\mu\, E = j\omega\mu\, J + \frac{1}{\varepsilon}\nabla\rho$$

$$\nabla^2 H + \omega^2 \varepsilon\mu\, H = -\nabla\times J$$

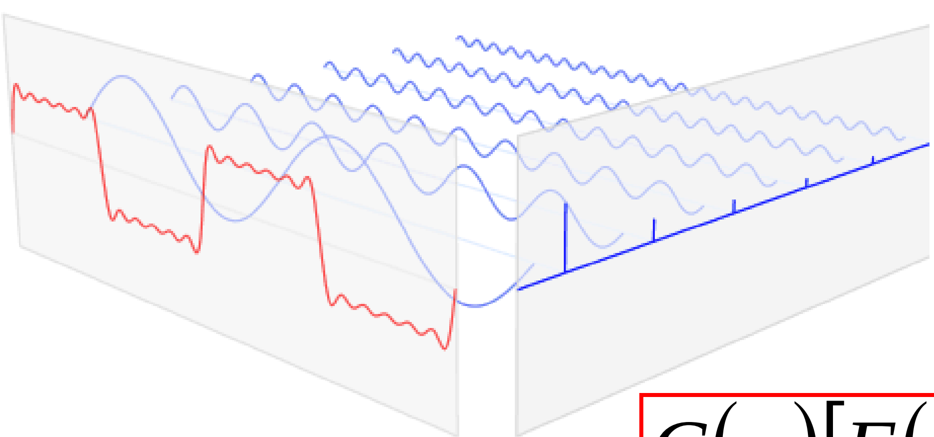
$$\nabla\cdot E = \frac{\rho}{\varepsilon}$$

$$\nabla\cdot H = 0$$

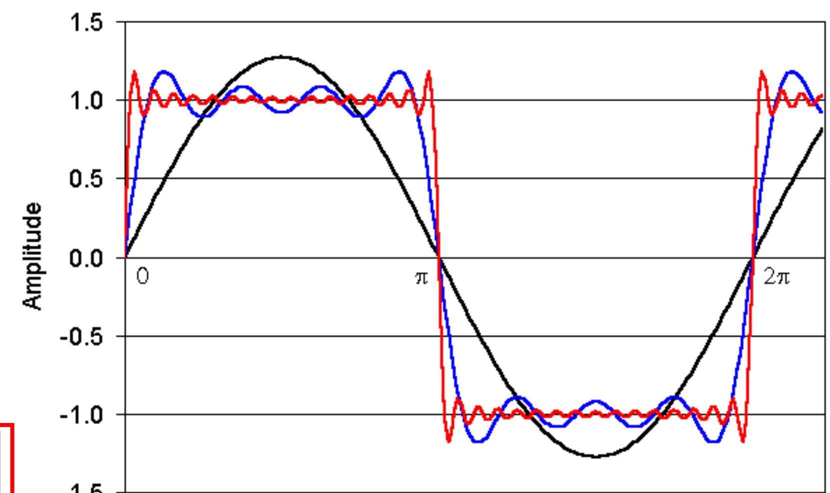
Mathematical models

- particular cases where analytical solution exists
  - harmonic signals, Fourier Transform, frequency spectrum

$$X = X_0 e^{j\cdot\omega\cdot t} \qquad \frac{\partial X}{\partial t} = j\cdot\omega\cdot X \qquad g(\omega) = \int_{-\infty}^{\infty} f(t)\cdot e^{-j\omega t}dt \qquad f(t) = \int_{-\infty}^{\infty} g(\omega)\cdot e^{j\omega t}d\omega$$

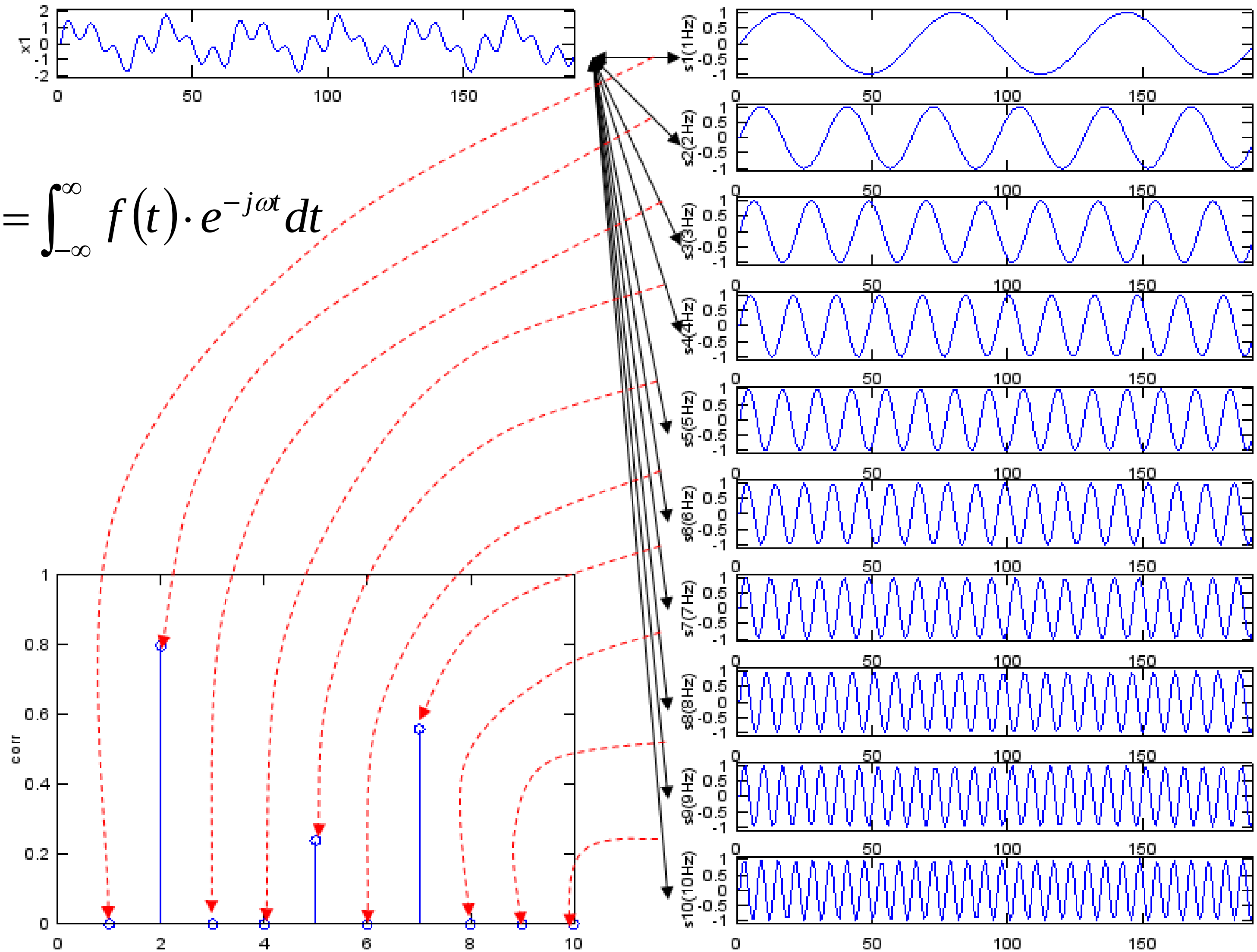


$$G(\omega)[F(\omega)]$$

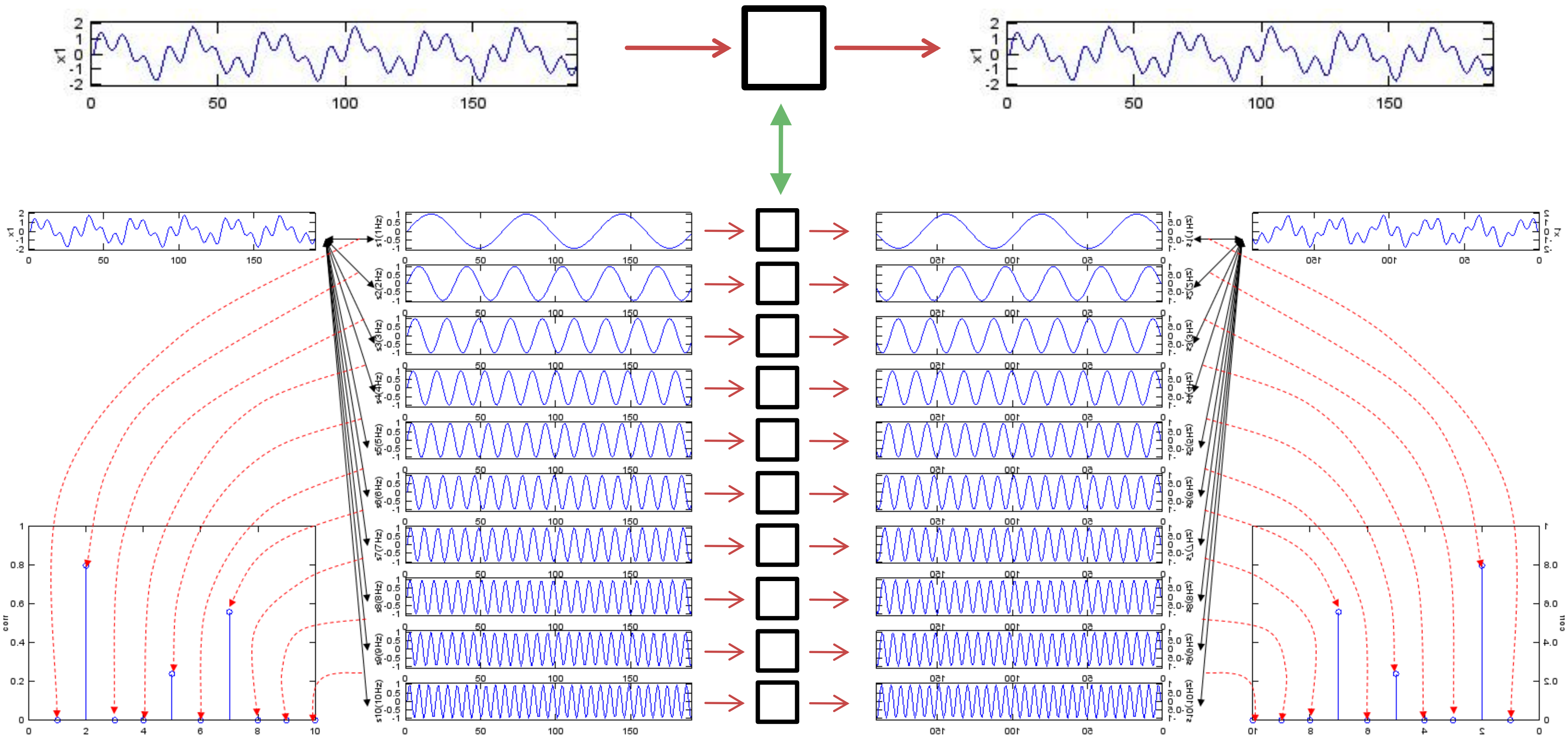


Mathematical models

$$g(\omega)=\int_{-\infty}^{\infty} f(t)\cdot e^{-j\omega t}dt$$



Mathematical models



$$F(\omega)=\int_{-\infty}^{\infty} f(t)\cdot e^{-j\omega t}dt$$

$$G(\omega)[F(\omega)]$$

$$g(t)=\int_{-\infty}^{\infty} G(\omega)\cdot e^{j\omega t}d\omega$$

Wave equations

- Helmoltz equations or Wave equations

Medium void of free electric charges

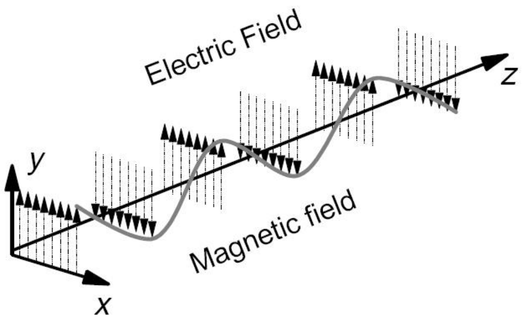
$$\nabla^2 E - \gamma^2 E = 0$$

$$\nabla^2 H - \gamma^2 H = 0$$

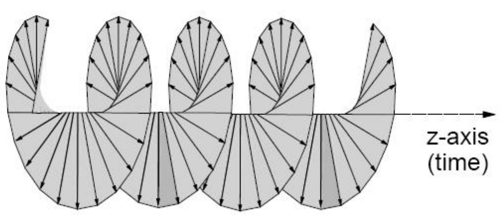
$$\gamma^2 = -\omega^2 \varepsilon\mu + j\omega\mu\sigma$$

γ – propagation constant (known also as phase constant or wave number)

Solutions of the wave equations



Wave Propagation



Circular Polarization

Electric field only in Oy direction, ← **through judicious choice**
wave traveling after Oz direction ← **of the coordinate system**

$$E_y = E_+ e^{-\gamma \cdot z} + E_- e^{\gamma \cdot z}$$

$$\gamma = \sqrt{-\omega^2 \varepsilon\mu + j\omega\mu\sigma} = \alpha + j \cdot \beta$$

If we have only the positive direction wave E_+=> A

$$E_y = Ae^{-(\alpha + j \cdot \beta) \cdot z}$$

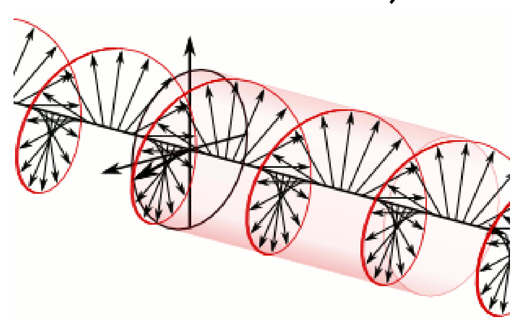
Harmonic Field

$$E_y = A \cdot e^{-\alpha \cdot z} \cdot e^{j(\omega \cdot t - \beta \cdot z)}$$

Amplitude

Attenuation

Wave Propagation (simultaneous space and time variation)



Attenuation

$$E_y(z_1) = Ct \cdot e^{-\alpha \cdot z_1} \cdot e^{j(\omega t - \beta \cdot z_1)} \qquad E_y(z_2) = Ct \cdot e^{-\alpha \cdot z_2} \cdot e^{j(\omega t - \beta \cdot z_2)}$$

$$W, P \sim \int E^2$$

$$A = \frac{P_2}{P_1} = \frac{Ct^2 \cdot e^{-2\alpha \cdot z_2}}{Ct^2 \cdot e^{-2\alpha \cdot z_1}} = e^{-2\alpha \cdot (z_2 - z_1)}$$

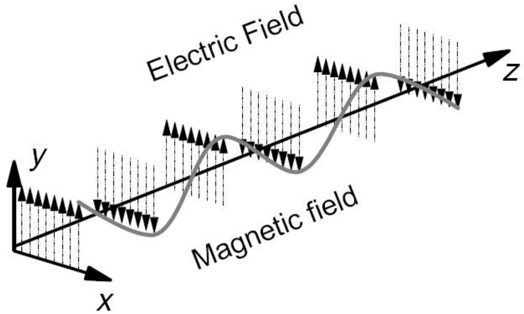
$$A[dB] = 10 \log_{10} \frac{P_2}{P_1} = 10 \log_{10} \left[e^{-2\alpha \cdot (z_2 - z_1)} \right]$$

$$A[dB] = -20 \cdot \alpha \cdot (z_2 - z_1) \log_{10} e = -8.686 \cdot \alpha \cdot (z_2 - z_1)$$

$$A / L[dB / km] = -8.686 \cdot \alpha < 0$$

- Attenuation usually expressed in **dB/km**
  - most of the time a positive value is used
  - "-" sign = **implied** by the word used

Plane wave parameters



$$\nabla \times E = -j\omega\mu \cdot H$$

$$H_x = \frac{j\gamma \cdot E_y}{\omega\mu}$$

Lossless Medium, $\sigma = 0$ $\qquad \gamma = j\omega \cdot \sqrt{\varepsilon\mu}$

$$\eta = \frac{E_y}{H_x} = \sqrt{\frac{\mu}{\varepsilon}}$$ intrinsic impedance of the medium

$$E_y = A \cdot e^{-\alpha \cdot z} \cdot e^{j(\omega \cdot t - \beta \cdot z)}$$

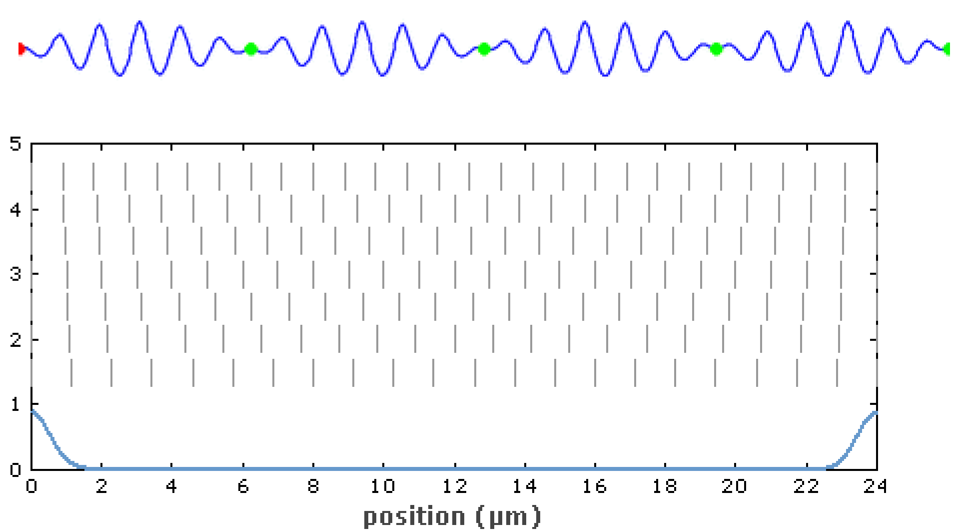
constant phase points: $(\omega \cdot t - \beta \cdot z) = \text{const}$

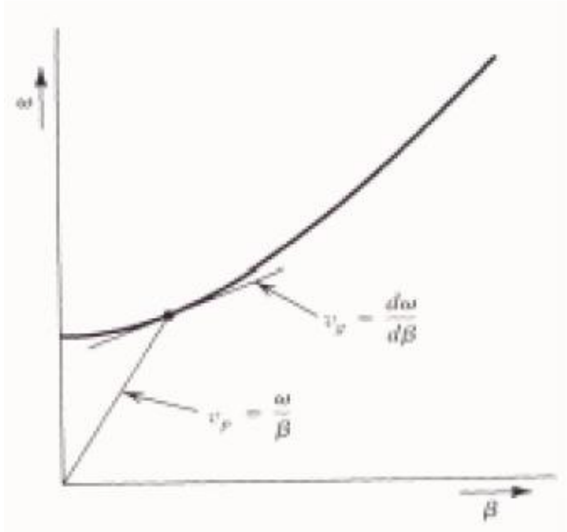
Phase velocity $\quad v_p = \frac{dz}{dt} = \frac{\omega}{\beta} = \frac{1}{\sqrt{\varepsilon\mu}}$

Group velocity $\quad v_g = \frac{dz}{dt} = \frac{d\omega}{d\beta}$ in dispersive media where $\beta = \beta(\omega)$

Group and phase velocities

- Phase velocity – **virtual** speed at which a constant phase point travels (in certain conditions might be greater than the speed of light)
- Group velocity – speed at which the signal (energy, information) propagates (always less or equal to the speed of light in that medium)





Plane wave parameters

- In vid

$$\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 377\Omega \qquad v = v_g = c_0 \qquad c_0 = \frac{1}{\sqrt{\varepsilon_0 \cdot \mu_0}} = 2{,}99790 \cdot 10^8 \; m/s$$

$$\lambda_0 = \frac{2\pi}{\beta} = \frac{c_0}{f} \qquad T = \frac{2\pi}{\omega} = \frac{1}{f}$$

Space periodicity — Time periodicity

- In mediu nedispersiv ε_r

$$c = \frac{1}{\sqrt{\varepsilon \cdot \mu_0}} = \frac{1}{\sqrt{\varepsilon_0 \varepsilon_r \cdot \mu_0}} = \frac{c_0}{\sqrt{\varepsilon_r}}$$

$n = \sqrt{\varepsilon_r}$ refractive index of a medium $\qquad c = \dfrac{c_0}{n}$

$$T = \frac{2\pi}{\omega} = \frac{1}{f} \qquad \lambda = \frac{2\pi}{\beta} = \frac{c}{f} \qquad \lambda = \frac{c_0}{\sqrt{\varepsilon_r} \cdot f} = \frac{\lambda_0}{\sqrt{\varepsilon_r}}$$

Solutions of the wave equations

$$E_y = E^+ e^{-\gamma \cdot z} + E^- e^{\gamma \cdot z}$$

Electric field only in Oy direction, ← through judicious choice
wave traveling after Oz direction ← of the coordinate system

$$\gamma = \sqrt{-\omega^2 \varepsilon \mu + j\omega\mu\sigma} = \alpha + j \cdot \beta$$

- wave
  - incident
  - reflected
- wave
  - direct
  - inverse

$$E_y = E^+ \cdot e^{-\alpha \cdot z} \cdot e^{j(\omega \cdot t - \beta \cdot z)}$$

$$(\omega \cdot t - \beta \cdot z) = \text{const}$$

$$E_y = E^- \cdot e^{\alpha \cdot z} \cdot e^{j(\omega \cdot t + \beta \cdot z)}$$

$$(\omega \cdot t + \beta \cdot z) = \text{const}$$

points of constant phase

Solutions of the wave equations

- wave
  - incident
  - reflected
- wave
  - direct
  - inverse

$$E_y = E^+ \cdot e^{-\alpha \cdot z} \cdot e^{j(\omega \cdot t - \beta \cdot z)} + E^- \cdot e^{-\alpha \cdot z} \cdot e^{j(\omega \cdot t + \beta \cdot z)}$$

$$H_z = H^+ \cdot e^{-\alpha \cdot z} \cdot e^{j(\omega \cdot t - \beta \cdot z)} + H^- \cdot e^{-\alpha \cdot z} \cdot e^{j(\omega \cdot t + \beta \cdot z)}$$

$$V(z) = V^+ \cdot e^{-\alpha \cdot z} \cdot e^{j(\omega \cdot t - \beta \cdot z)} + V^- \cdot e^{-\alpha \cdot z} \cdot e^{j(\omega \cdot t + \beta \cdot z)}$$

$$I(z) = I^+ \cdot e^{-\alpha \cdot z} \cdot e^{j(\omega \cdot t - \beta \cdot z)} + I^- \cdot e^{-\alpha \cdot z} \cdot e^{j(\omega \cdot t + \beta \cdot z)}$$

$$V(z) = V^+ \cdot e^{j(\omega \cdot t - \beta \cdot z)} + V^- \cdot e^{j(\omega \cdot t + \beta \cdot z)}$$

Modes in delimited media

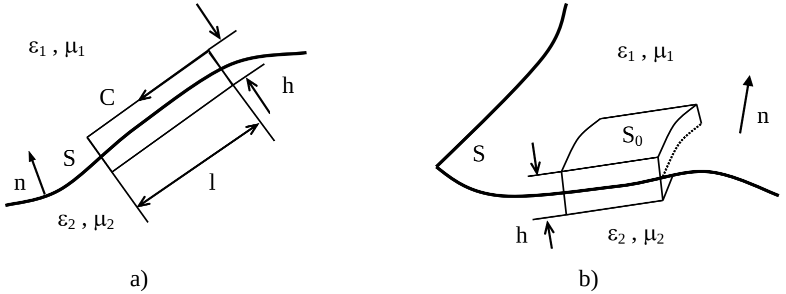
- Electromagnetic fields with harmonic time dependence
  - Maxwell's Equations simplified

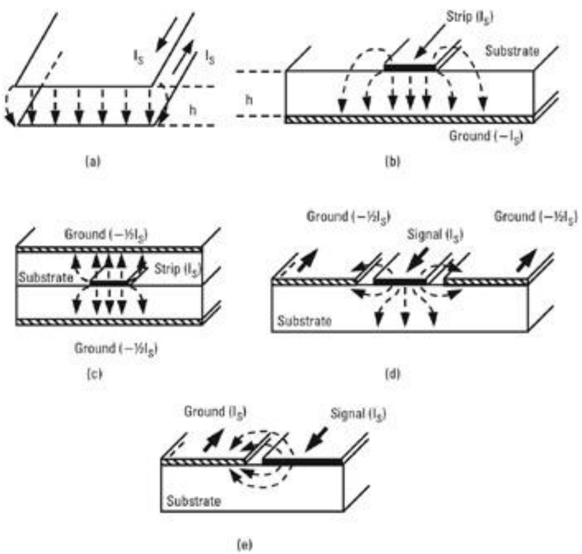
$$X = X_0 e^{j \cdot \omega \cdot t} \qquad \frac{\partial X}{\partial t} = j \cdot \omega \cdot X \qquad g(\omega) = \int_{-\infty}^{\infty} f(t) \cdot e^{-j\omega t} dt \qquad f(t) = \int_{-\infty}^{\infty} g(\omega) \cdot e^{j\omega t} d\omega$$

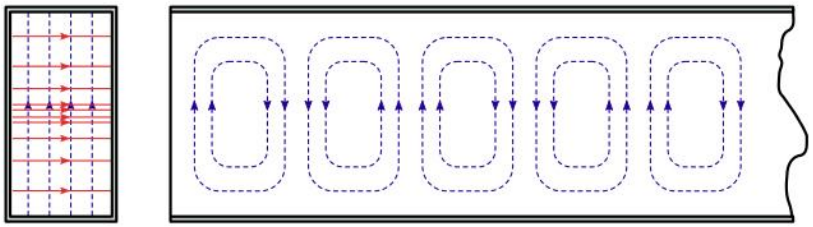
- In delimited media the solutions of Maxwell's Equations must also verify boundary conditions
  - solutions must respect some supplemental conditions

Modes in delimited media

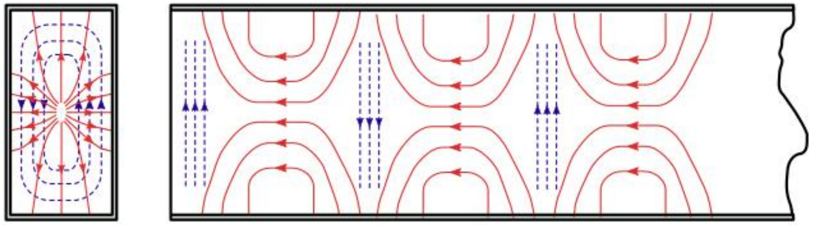
- Electric field must always be normal on an electric wall or annulled
- Magnetic field must always be tangent to an electric wall or annulled





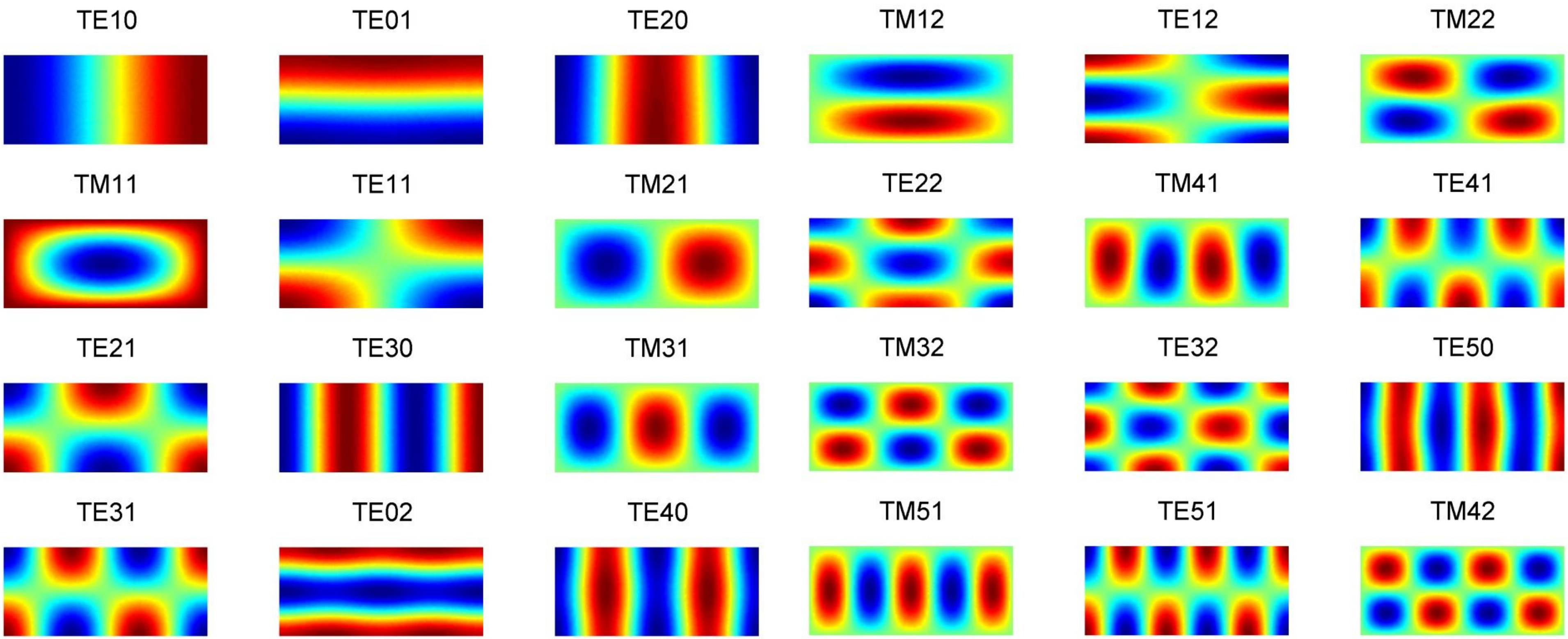


TE_{10}



TM_{11}

Moduri in medii delimitate



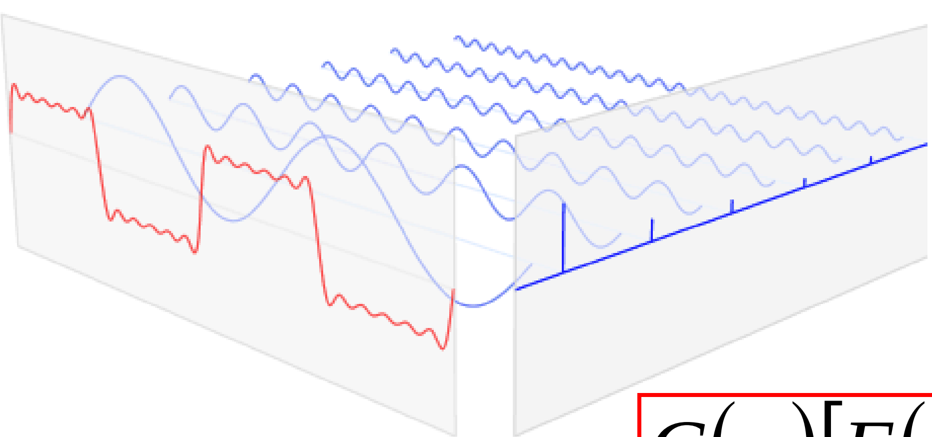
- Similar with Fourier Transform $g(\omega)=\int_{-\infty}^{\infty} f(t)\cdot e^{-j\omega t}dt \quad f(t)=\int_{-\infty}^{\infty} g(\omega)\cdot e^{j\omega t}d\omega$

$$E^+, E^- = \sum_{1}^{\infty} A_i \cdot Mod_i \qquad A_i = \langle E, Mod_i \rangle$$

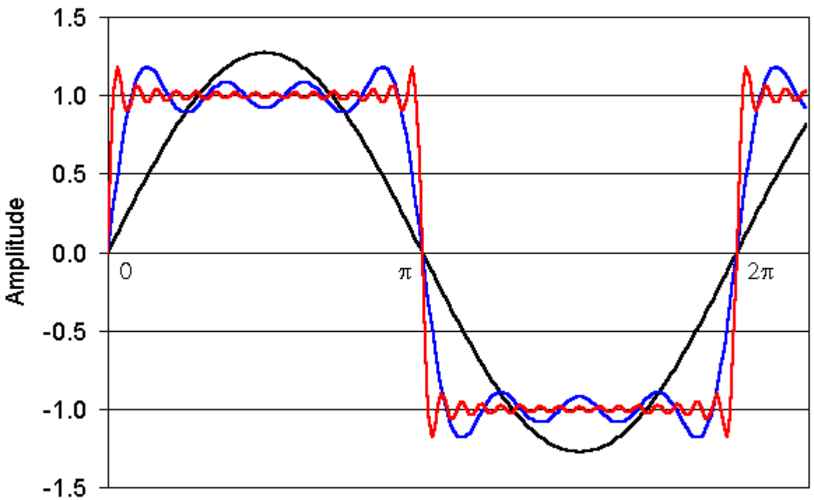
Mathematical modeling

- particular cases where analytical solution exists
  - harmonic signals, Fourier Transform, frequency spectrum

$$X = X_0 e^{j\cdot\omega\cdot t} \qquad \frac{\partial X}{\partial t} = j\cdot\omega\cdot X \qquad g(\omega) = \int_{-\infty}^{\infty} f(t)\cdot e^{-j\omega t}\,dt \qquad f(t) = \int_{-\infty}^{\infty} g(\omega)\cdot e^{j\omega t}\,d\omega$$

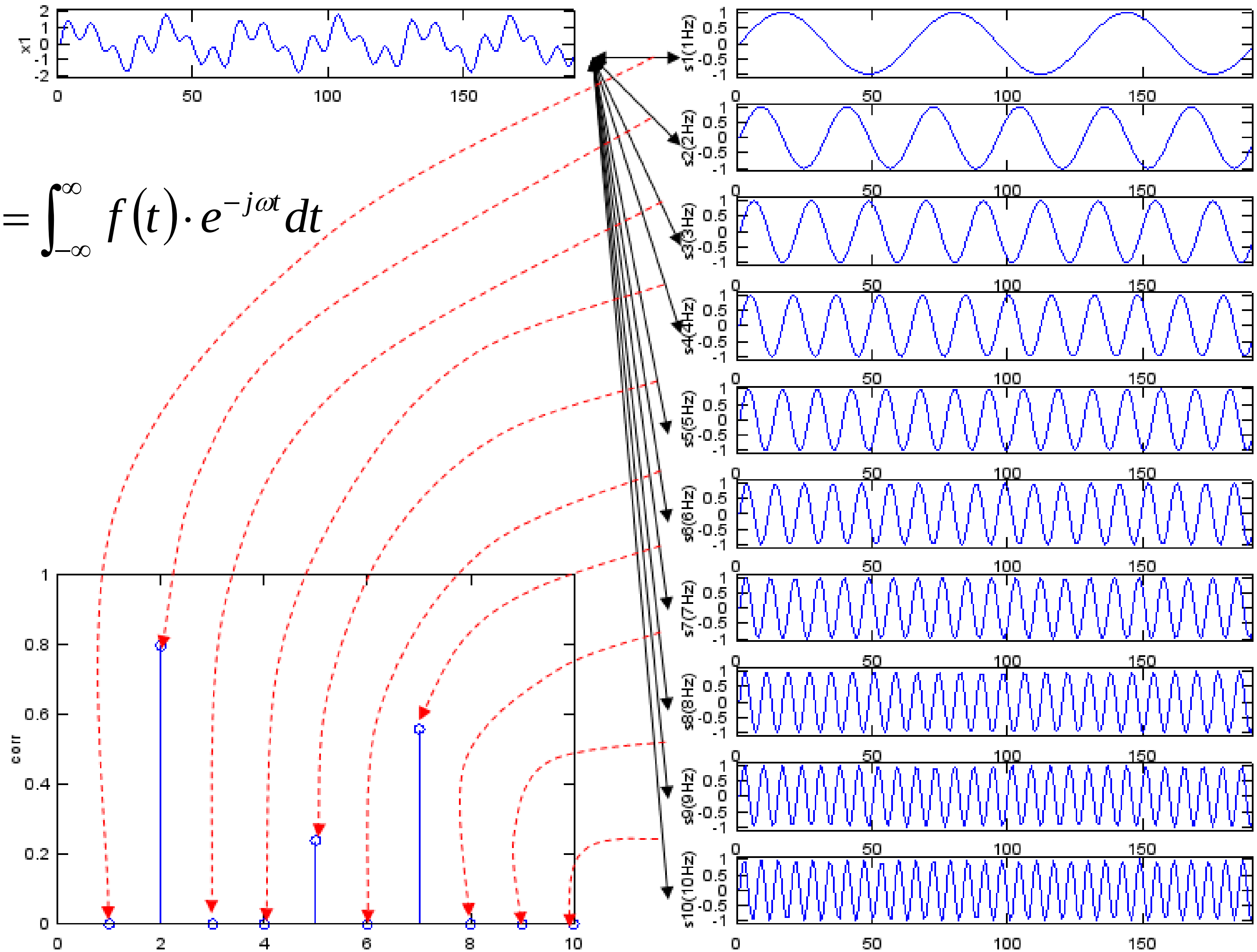


$$G(\omega)[F(\omega)]$$

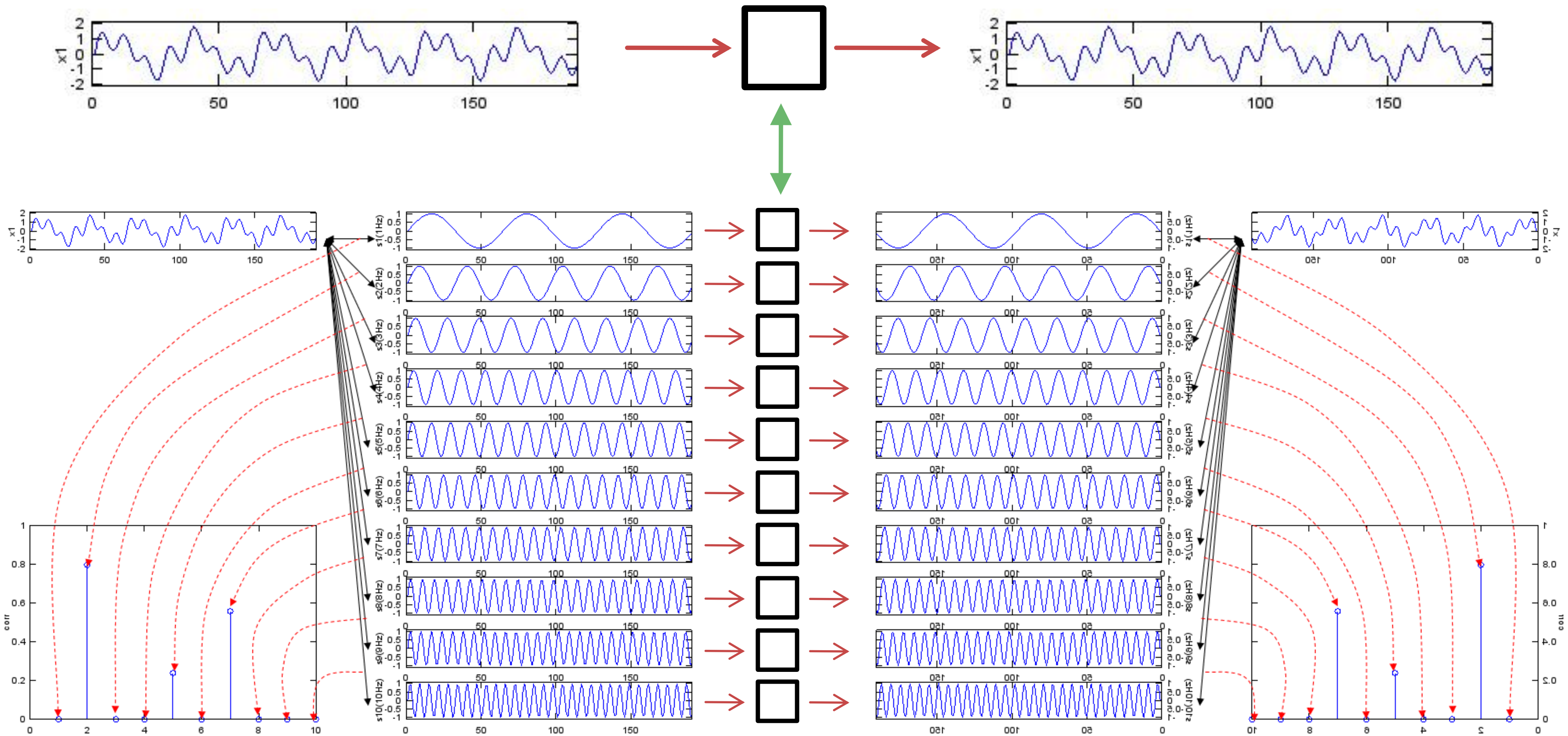


Mathematical modeling

$$g(\omega)=\int_{-\infty}^{\infty} f(t)\cdot e^{-j\omega t}dt$$



Mathematical modeling



$$F(\omega)=\int_{-\infty}^{\infty} f(t)\cdot e^{-j\omega t}dt$$

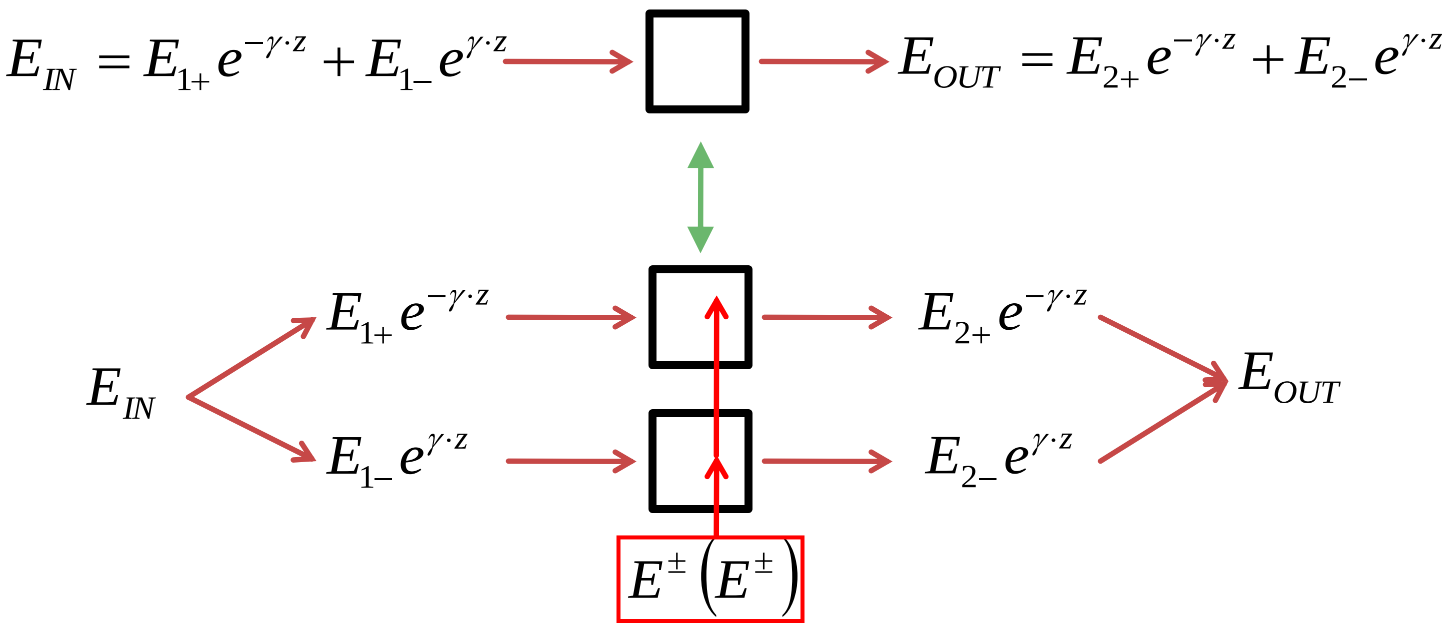
$$G(\omega)[F(\omega)]$$

$$g(t)=\int_{-\infty}^{\infty} G(\omega)\cdot e^{j\omega t}d\omega$$

Mathematical modeling

- particular cases where analytical solution exists
  - wave in a single direction $E^{+}\left(E^{+}\right), \quad E^{-}\left(E^{-}\right)$
  - wave
    - incident
    - reflected
  - wave
    - direct
    - inverse

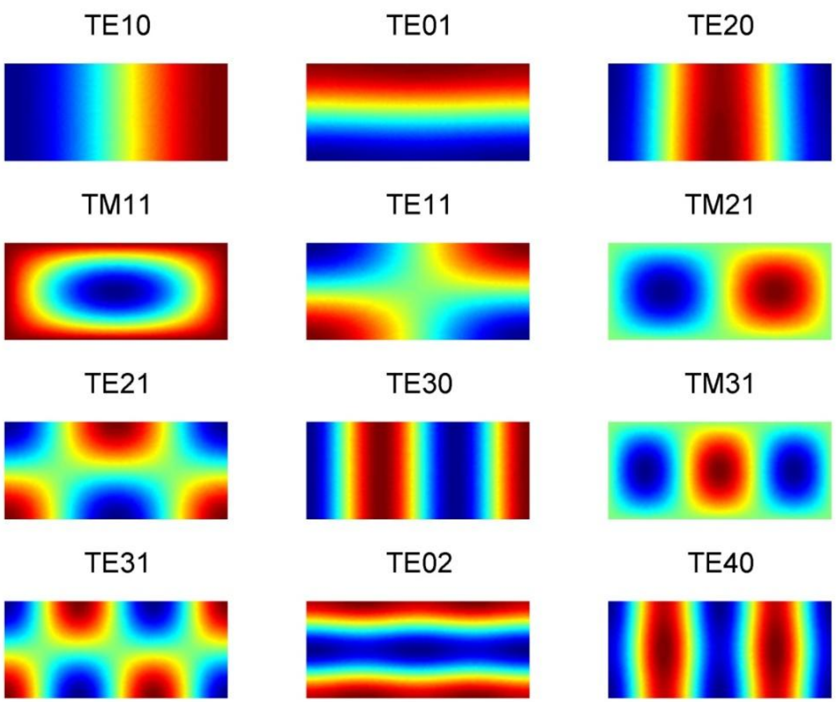
$$E_y = E^{+} \cdot e^{-\alpha \cdot z} \cdot e^{j(\omega t - \beta \cdot z)} + E^{-} \cdot e^{-\alpha \cdot z} \cdot e^{j(\omega t + \beta \cdot z)}$$



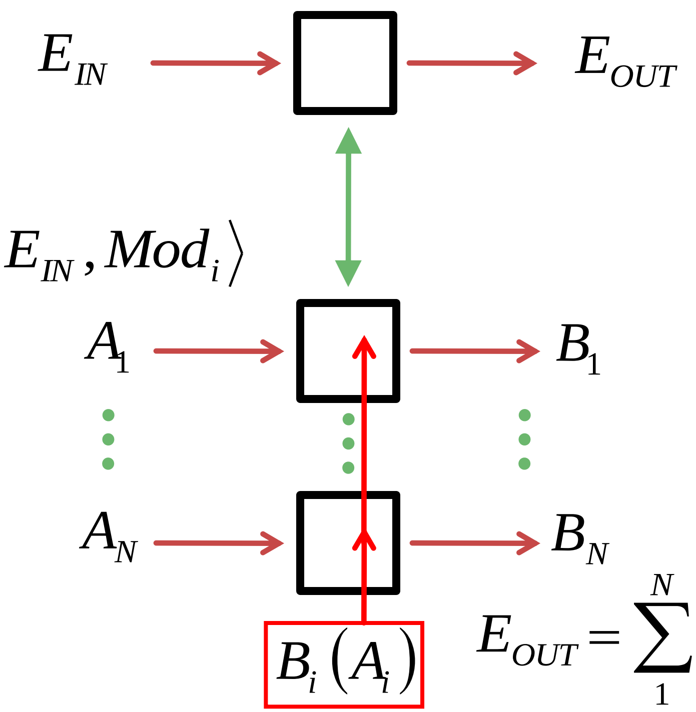
Mathematical modeling

- particular cases where analytical solution exists
  - modes in delimited media $B_i(A_i)$

$$E = \sum_{1}^{\infty} A_i \cdot Mod_i \qquad A_i = \langle E, Mod_i \rangle$$



$$A_i = \langle E_{IN}, Mod_i \rangle$$

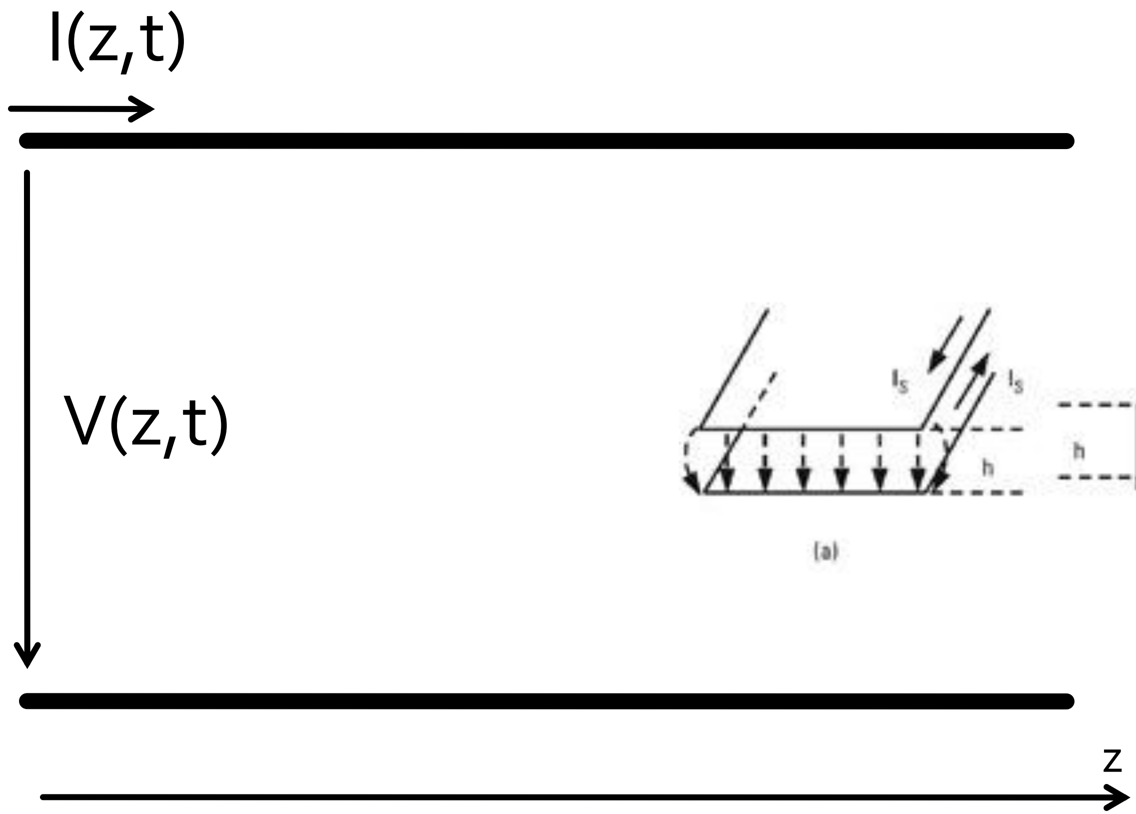


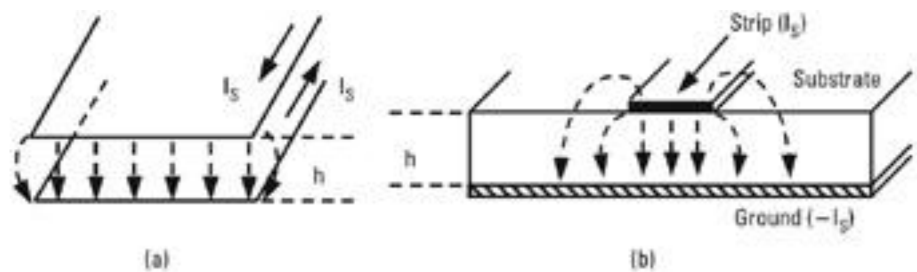
$$E_{OUT} = \sum_{1}^{N} B_i \cdot Mod_i$$

TEM transmission lines

Transmission line

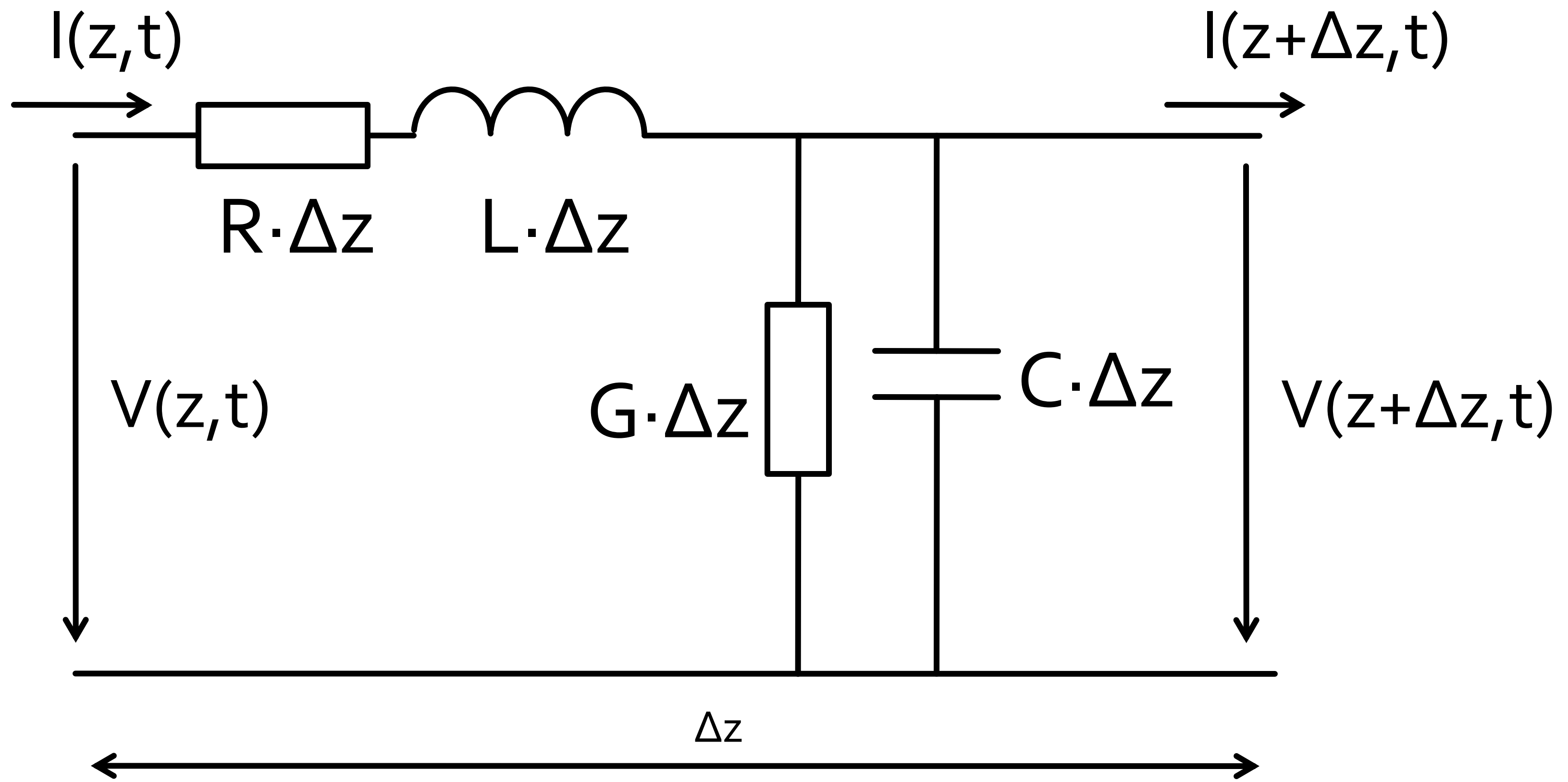
- TEM wave propagation, at least two conductors





Transmission line equivalent model

- TEM wave propagation, at least two conductors



Telegrapher equations

- time domain

$$\frac{\partial v(z,t)}{\partial z} = -R \cdot i(z,t) - L \cdot \frac{\partial i(z,t)}{\partial t} \qquad \text{K II}$$

$$\frac{\partial i(z,t)}{\partial z} = -G \cdot v(z,t) - C \cdot \frac{\partial v(z,t)}{\partial t} \qquad \text{K I}$$

- harmonic signals

$$\frac{dV(z)}{dz} = -(R + j \cdot \omega \cdot L) \cdot I(z)$$

$$\frac{dI(z)}{dz} = -(G + j \cdot \omega \cdot C) \cdot V(z)$$

$$\Big/ \frac{d}{dz}(\ldots)$$

Solution

$$\frac{d^2 V(z)}{dz^2} - \gamma^2 \cdot V(z) = 0$$

$$\gamma = \alpha + j \cdot \beta = \sqrt{(R + j \cdot \omega \cdot L) \cdot (G + j \cdot \omega \cdot C)}$$

$$\frac{d^2 I(z)}{dz^2} - \gamma^2 \cdot I(z) = 0$$

$$\nabla^2 E - \gamma^2 E = 0$$

$$\nabla^2 H - \gamma^2 H = 0$$

$$E_y = E_+ e^{-\gamma \cdot z} + E\_ e^{\gamma \cdot z}$$

$$\gamma^2 = -\omega^2 \varepsilon\mu + j\omega\mu\sigma$$

Solutions

$$\begin{cases} V(z) = V_0^+ e^{-\gamma \cdot z} + V_0^- e^{\gamma \cdot z} \\ I(z) = I_0^+ e^{-\gamma \cdot z} + I_0^- e^{\gamma \cdot z} \end{cases}$$

$$\gamma = \alpha + j \cdot \beta = \sqrt{(R + j \cdot \omega \cdot L) \cdot (G + j \cdot \omega \cdot C)}$$

$$V(z) = V_0^+ e^{-\gamma \cdot z} + V_0^- e^{\gamma \cdot z}$$

$$\frac{dV(z)}{dz} = -(R + j \cdot \omega \cdot L) \cdot I(z)$$

$$I(z) = \frac{\gamma}{R + j \cdot \omega \cdot L} \left(V_0^+ e^{-\gamma \cdot z} - V_0^- e^{\gamma \cdot z} \right)$$

$$Z_0 \equiv \frac{R + j \cdot \omega \cdot L}{\gamma} = \sqrt{\frac{R + j \cdot \omega \cdot L}{G + j \cdot \omega \cdot C}}$$

- Characteristic impedance of the line

$$\frac{V_0^+}{I_0^+} = Z_0 = -\frac{V_0^-}{I_0^-}$$

$$\lambda = \frac{2\pi}{\beta} \qquad v_f = \frac{\omega}{\beta} = \lambda \cdot f$$

The lossless line

- R=G=0

$$\gamma = \alpha + j\cdot\beta = \sqrt{(R + j\cdot\omega\cdot L)\cdot(G + j\cdot\omega\cdot C)} = j\cdot\omega\cdot\sqrt{L\cdot C}$$

$$\alpha = 0 \quad ; \quad \beta = \omega\cdot\sqrt{L\cdot C}$$

$$Z_0 = \sqrt{\frac{R + j\cdot\omega\cdot L}{G + j\cdot\omega\cdot C}} = \sqrt{\frac{L}{C}}$$

- Z_0 is **real**

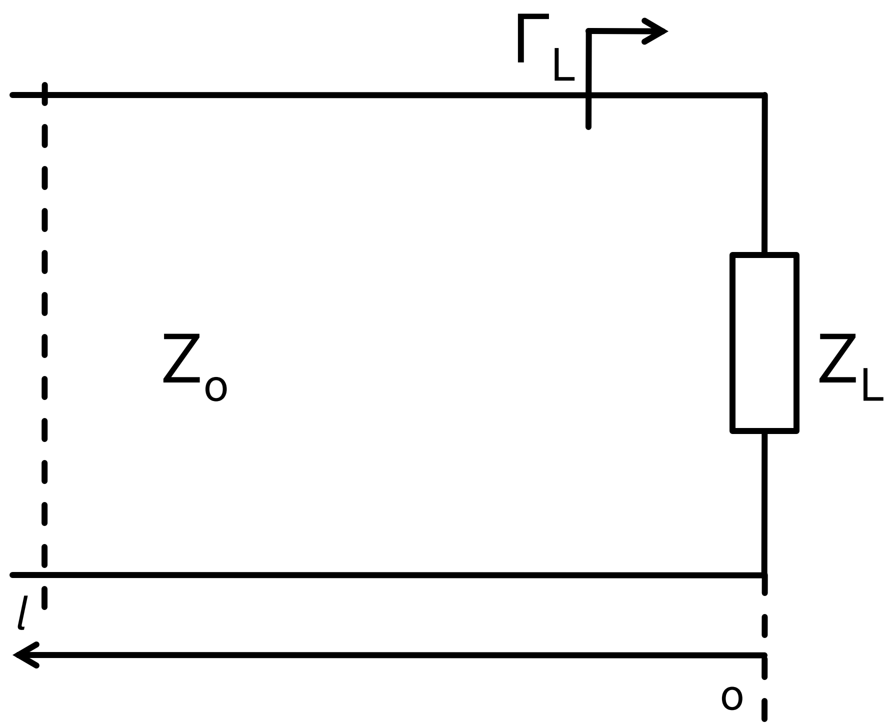
$$V(z) = V_0^+ e^{-j\cdot\beta\cdot z} + V_0^- e^{j\cdot\beta\cdot z}$$

$$I(z) = \frac{V_0^+}{Z_0} e^{-j\cdot\beta\cdot z} - \frac{V_0^-}{Z_0} e^{j\cdot\beta\cdot z}$$

$$\lambda = \frac{2\pi}{\omega\cdot\sqrt{LC}}$$

$$v_f = \frac{1}{\sqrt{LC}}$$

The lossless line



$$V(z) = V_0^+ e^{-j\cdot\beta\cdot z} + V_0^- e^{j\cdot\beta\cdot z}$$

$$I(z) = \frac{V_0^+}{Z_0} e^{-j\cdot\beta\cdot z} - \frac{V_0^-}{Z_0} e^{j\cdot\beta\cdot z}$$

$$Z_L = \frac{V(0)}{I(0)} \qquad Z_L = \frac{V_0^+ + V_0^-}{V_0^+ - V_0^-} \cdot Z_0$$

- voltage reflection coefficient

$$\Gamma = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

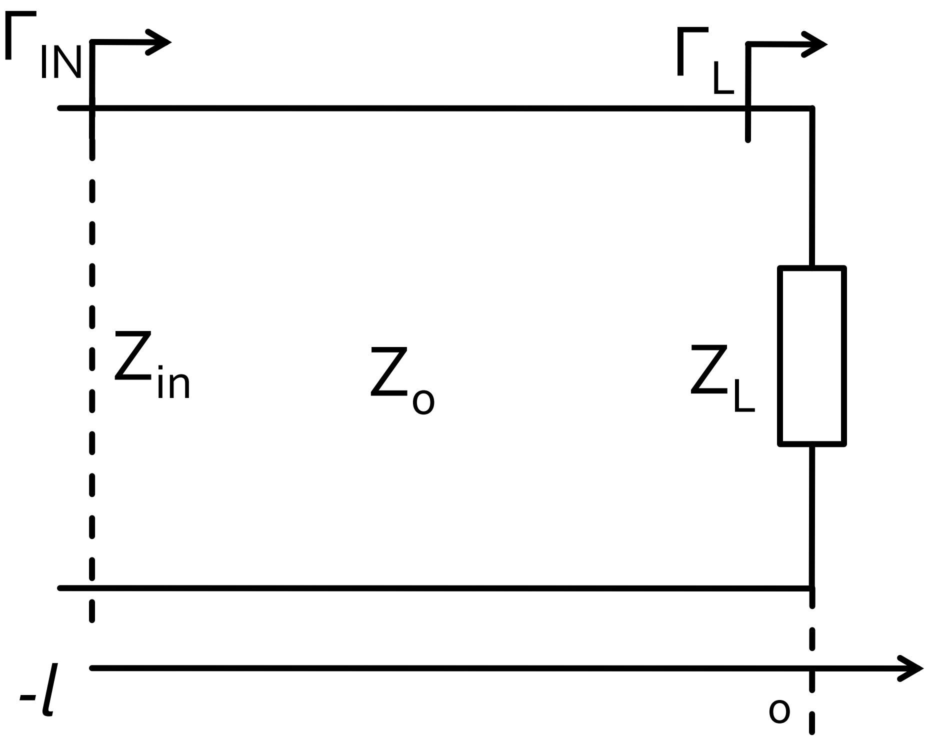
- Z_o real

The lossless line

- voltage reflection coefficient seen at the input of the line

$$V(z) = V_0^+ e^{-j\cdot\beta\cdot z} + V_0^- e^{j\cdot\beta\cdot z}$$

$$\Gamma = \Gamma(z) = \frac{V_0^-(z)}{V_0^+(z)}$$



$$V(0) = V_0^+ + V_0^- \qquad \Gamma(0) = \Gamma_L = \frac{V_0^-}{V_0^+}$$

$$V(-l) = V_0^+ e^{j\cdot\beta\cdot l} + V_0^- e^{-j\cdot\beta\cdot l}$$

$$\Gamma(-l) = \Gamma_{IN} = \frac{V_0^- \cdot e^{-j\cdot\beta\cdot l}}{V_0^+ \cdot e^{j\cdot\beta\cdot l}} = \Gamma(0)\cdot e^{-2j\cdot\beta\cdot l}$$

$$|\Gamma(-l)| = |\Gamma(0)|\cdot\left|e^{-2j\cdot\beta\cdot l}\right| = |\Gamma(0)|$$

$$\boxed{\Gamma(-l) = \Gamma(0)\cdot e^{-2j\cdot\beta\cdot l}}$$

The lossless line

$$V(z) = V_0^+ \cdot \left(e^{-j\cdot\beta\cdot z} + \Gamma \cdot e^{j\cdot\beta\cdot z}\right) \qquad I(z) = \frac{V_0^+}{Z_0} \cdot \left(e^{-j\cdot\beta\cdot z} - \Gamma \cdot e^{j\cdot\beta\cdot z}\right)$$

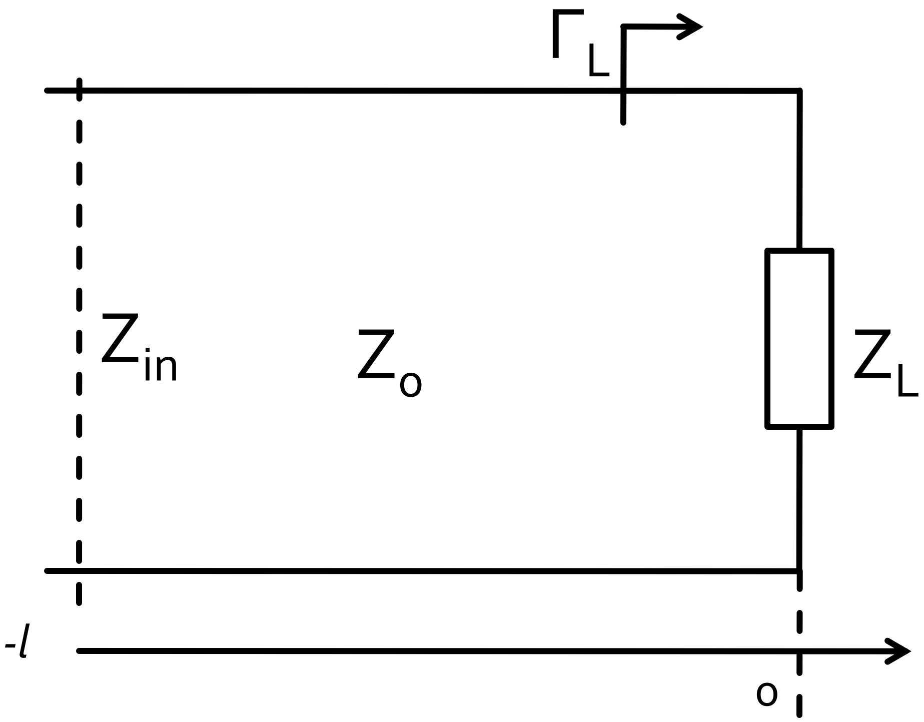
- time-average Power flow along the line

$$P_{\rm avg} = \frac{1}{2}{\rm Re}\left\{V(z)I(z)^*\right\} = \frac{1}{2}\frac{|V_o^+|^2}{Z_0}{\rm Re}\left\{1 - \Gamma^* e^{-2j\beta z} + \Gamma e^{2j\beta z} - |\Gamma|^2\right\}$$

$$P_{\rm avg} = \frac{1}{2}\frac{|V_o^+|^2}{Z_0}\left(1 - |\Gamma|^2\right)$$

- Total power delivered to the load = Incident power – “Reflected” power
- Return “Loss” [dB] $\quad {\rm RL} = -20 \log |\Gamma|\ {\rm dB}.$

The lossless line



$$V(-l) = V_0^+ e^{j\cdot\beta\cdot l} + V_0^- e^{-j\cdot\beta\cdot l}$$

$$I(-l) = \frac{V_0^+}{Z_0} e^{j\cdot\beta\cdot l} - \frac{V_0^-}{Z_0} e^{-j\cdot\beta\cdot l}$$

$$Z_{in} = \frac{V(-l)}{I(-l)} \qquad Z_{in} = Z_0 \cdot \frac{1+\Gamma\cdot e^{-2j\cdot\beta\cdot l}}{1-\Gamma\cdot e^{-2j\cdot\beta\cdot l}}$$

- the **input impedance** seen looking toward the load

$$Z_{in} = Z_0 \cdot \frac{(Z_L + Z_0)\cdot e^{j\cdot\beta\cdot l} + (Z_L - Z_0)\cdot e^{-j\cdot\beta\cdot l}}{(Z_L + Z_0)\cdot e^{j\cdot\beta\cdot l} - (Z_L - Z_0)\cdot e^{-j\cdot\beta\cdot l}}$$

$$Z_{in} = Z_0 \cdot \frac{Z_L + j\cdot Z_0\cdot\tan(\beta\cdot l)}{Z_0 + j\cdot Z_L\cdot\tan(\beta\cdot l)}$$

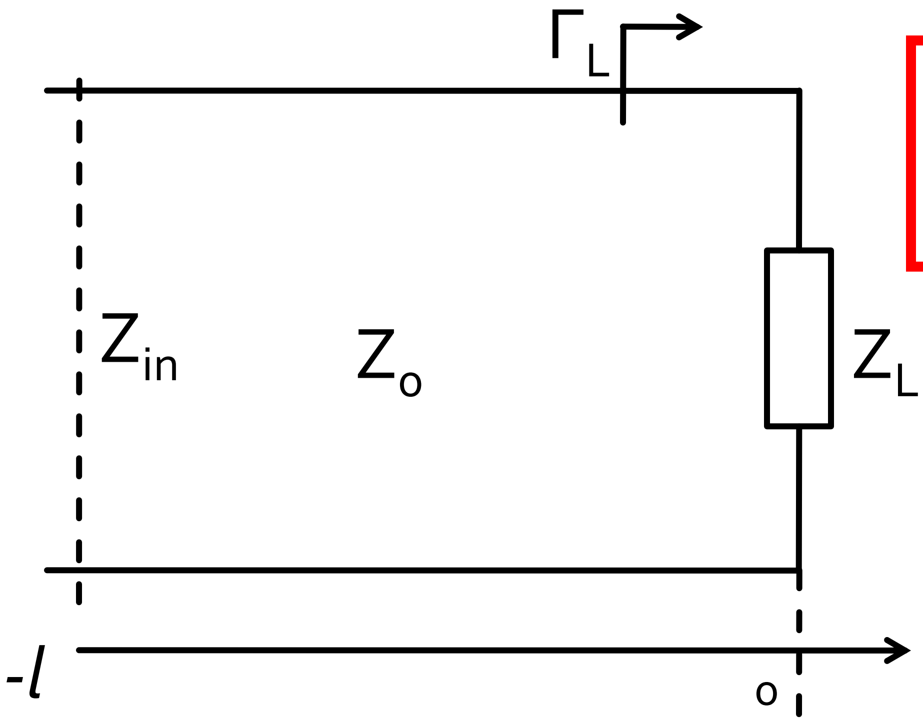
The lossless line

- the **input impedance** seen looking toward the load

$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}$$

The lossless line

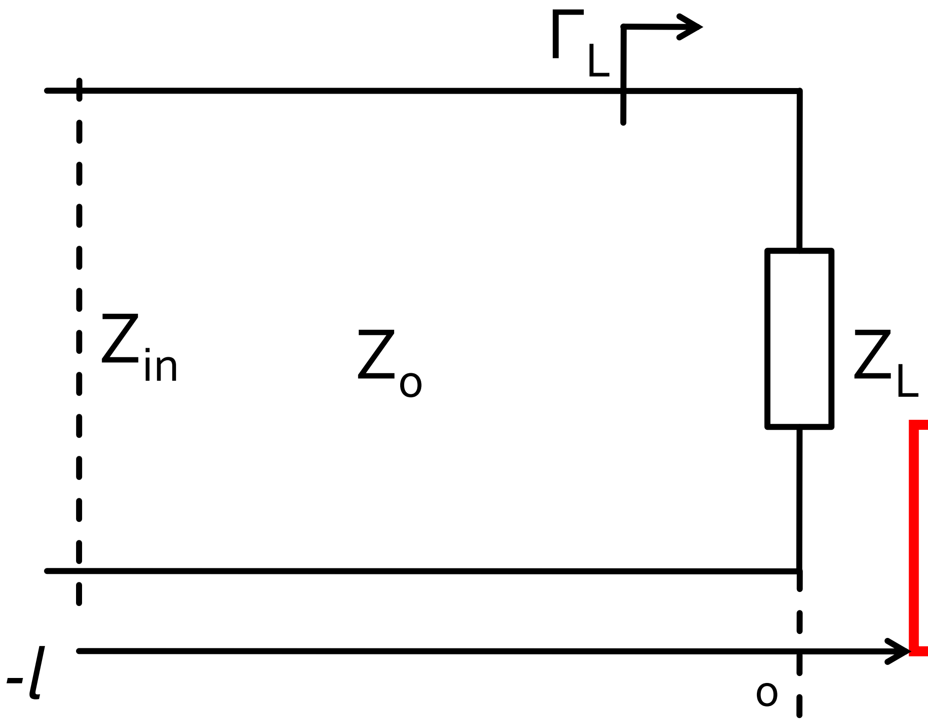
- input impedance of a length ***l*** of transmission line with characteristic impedance Z_o, loaded with an arbitrary impedance Z_L



$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}$$

The lossless line

- input impedance is **frequency dependent** through $\beta \cdot l$



$$v_f = \frac{\omega}{\beta} = \lambda \cdot f \qquad \lambda = \frac{2\pi}{\beta}$$

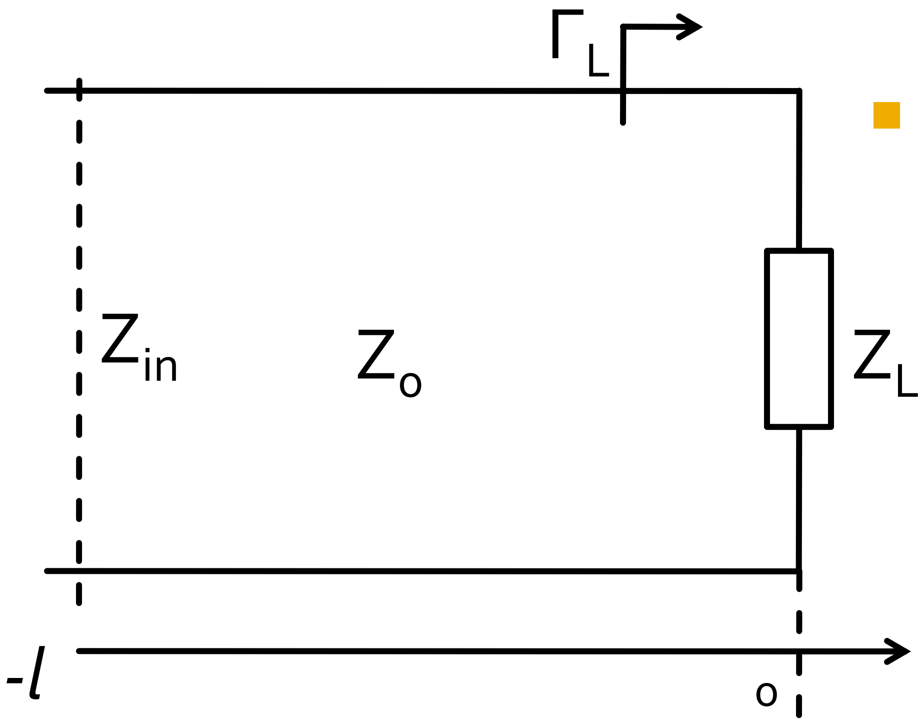
$$\beta \cdot l = \frac{2\pi}{\lambda} \cdot l = \frac{2\pi \cdot f}{v_f} \cdot l = \frac{2\pi \cdot l}{v_f} \cdot f$$

frequency dependence is periodical, imposed by the tan trigonometric function

$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}$$

The lossless line, special cases

- l = k·λ/2 $\beta \cdot l = \frac{2\pi}{\lambda} \cdot l = k \cdot \pi$ $\tan \beta \cdot l = 0$ $Z_{in} = Z_0$
- l = λ/4 + k·λ/2 $\tan \beta \cdot l \to \infty$ $Z_{in} = \frac{Z_0^2}{Z_L}$



- quarter-wave transformer

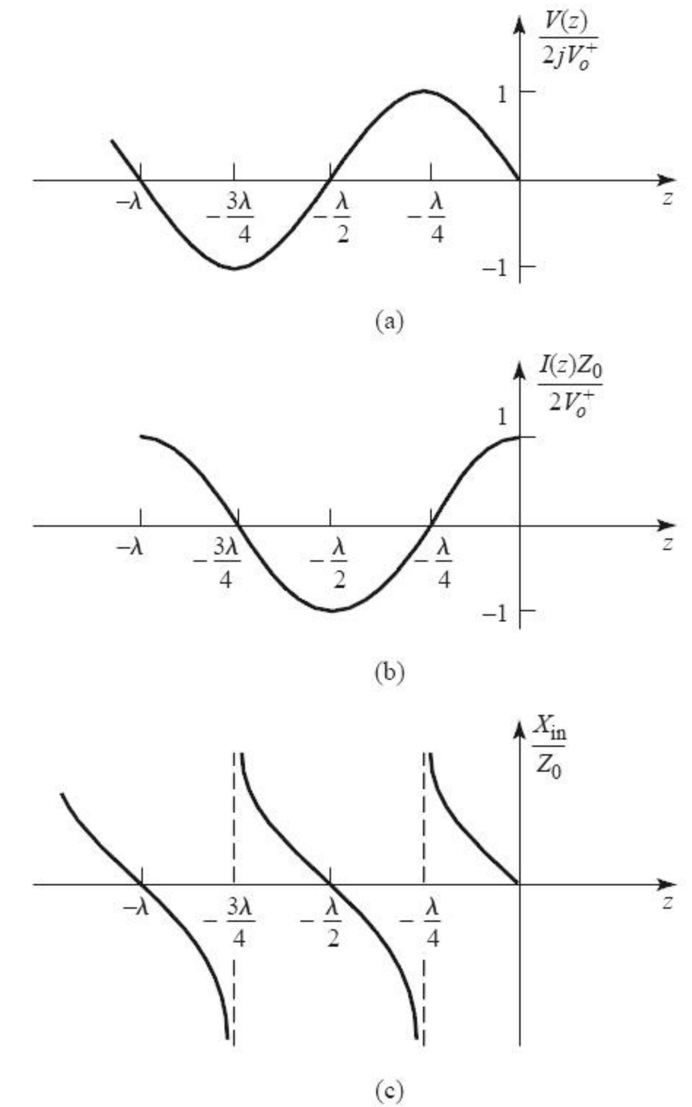
$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}$$

Short-circuited transmission line

- $Z_L = 0$
- purely imaginary for any length *l*
  - +/- → depending on *l* value

$$Z_{in} = j \cdot Z_0 \cdot \tan \beta \cdot l$$

$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan(\beta \cdot l)}{Z_0 + j \cdot Z_L \cdot \tan(\beta \cdot l)}$$

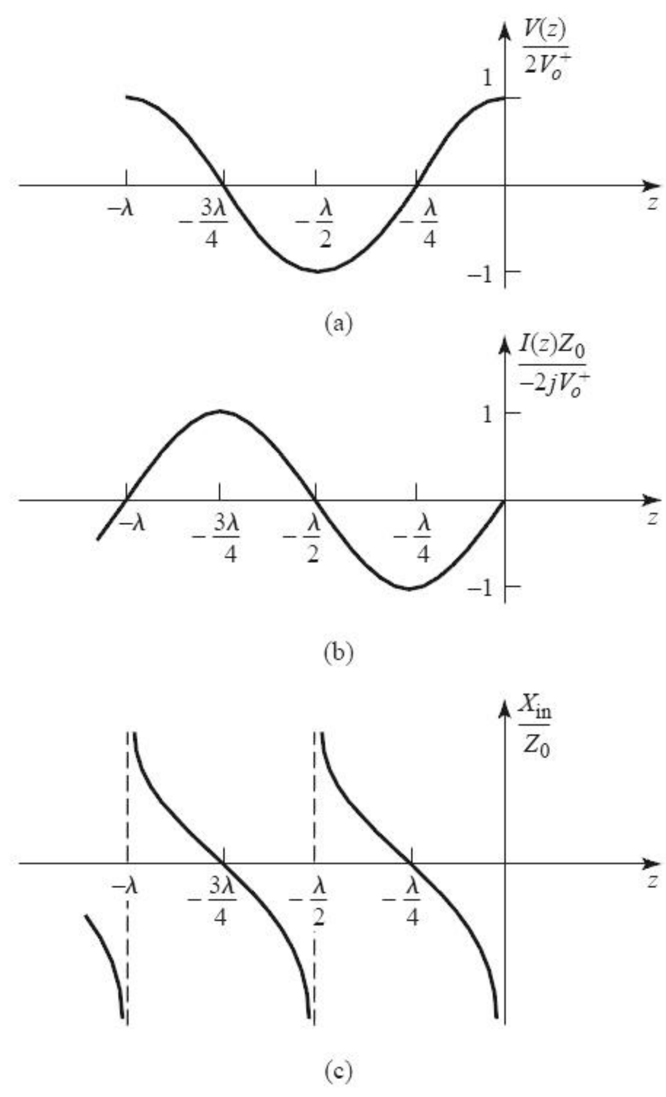


Open-circuited transmission line

- $Z_L = \infty \longrightarrow 1/Z_L = 0$
- purely imaginary for any length *l*
  - +/- → depending on *l* value

$$Z_{in} = -j \cdot Z_0 \cdot \cot \beta \cdot l$$

$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan(\beta \cdot l)}{Z_0 + j \cdot Z_L \cdot \tan(\beta \cdot l)}$$



Voltage standing wave ratio

$$V(z)=V_0^+\cdot\left(e^{-j\cdot\beta\cdot z}+\Gamma\cdot e^{j\cdot\beta\cdot z}\right)\quad |V(z)|=\left|V_0^+\right|\cdot\left|e^{-j\cdot\beta\cdot z}\right|\cdot\left|1+\Gamma\cdot e^{2j\cdot\beta\cdot z}\right|\quad \Gamma=|\Gamma|\cdot e^{j\theta}$$

$$|V(z)|=\left|V_0^+\right|\cdot\left|1+|\Gamma|\cdot e^{\theta+2j\cdot\beta\cdot z}\right|$$

maximum magnitude value for $e^{\theta+2j\cdot\beta\cdot z}=1$ $\quad V_{max}=\left|V_0^+\right|\cdot\left(1+|\Gamma|\right)$

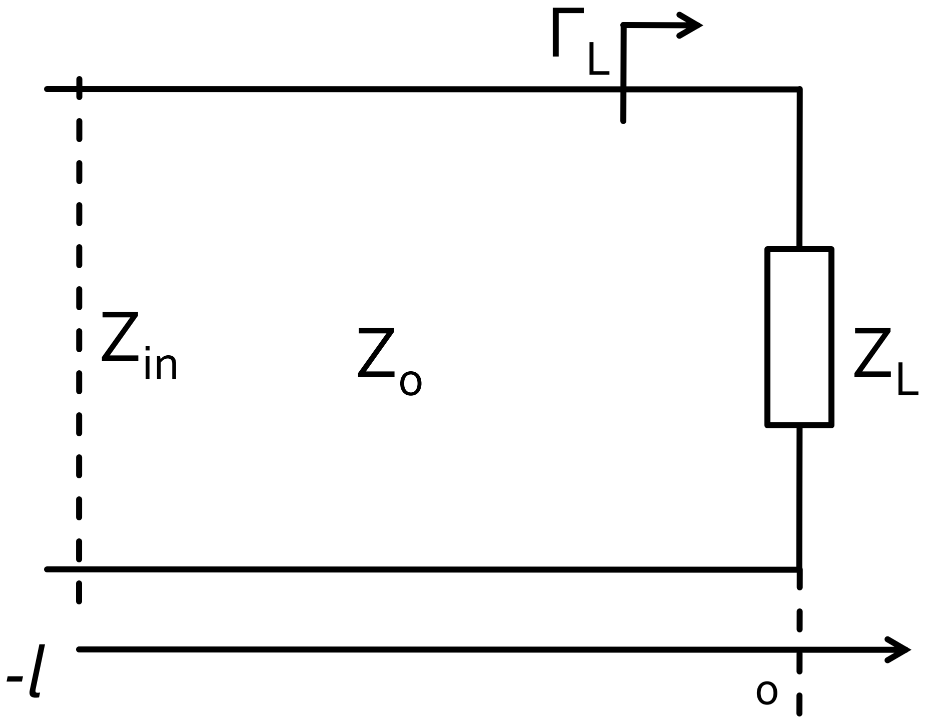
minimum magnitude value for $e^{\theta+2j\cdot\beta\cdot z}=-1$ $\quad V_{min}=\left|V_0^+\right|\cdot\left(1-|\Gamma|\right)$

- SWR is defined as the ratio between maximum and minimum
  - (Voltage) Standing Wave Ratio

$$VSWR=\frac{V_{max}}{V_{min}}=\frac{1+|\Gamma|}{1-|\Gamma|}$$

  - real number $1 \le VSWR < \infty$
  - a measure of the mismatch (SWR = 1 means a matched line)

The lossless line +/-



$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}$$

$$V(z) = V_0^+ e^{-\gamma \cdot z} + V_0^- e^{\gamma \cdot z}$$

$$I(z) = I_0^+ e^{-\gamma \cdot z} + I_0^- e^{\gamma \cdot z}$$

$$\Gamma(-l) = \Gamma(0) \cdot e^{-2j \cdot \beta \cdot l}$$

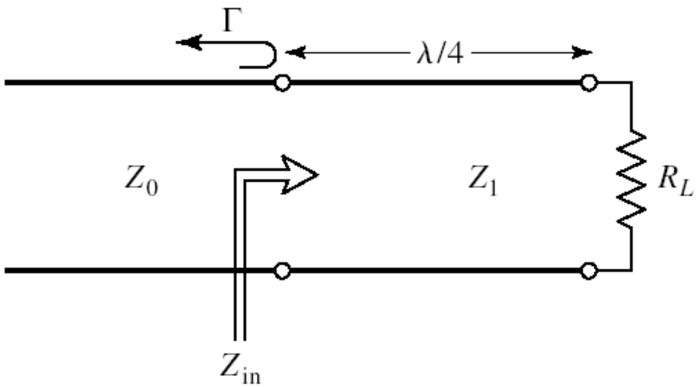
$$\Gamma_{in} = \Gamma_L \cdot e^{-2j \cdot \beta \cdot l}$$

Impedance Matching

Laboratory no. 1

The quarter-wave transformer

- Feed line – input line with characteristic impedance Z_o
- Real load impedance R_L
- We desire matching the load to the fider with a second line with the length λ/4 and characteristic impedance Z_1

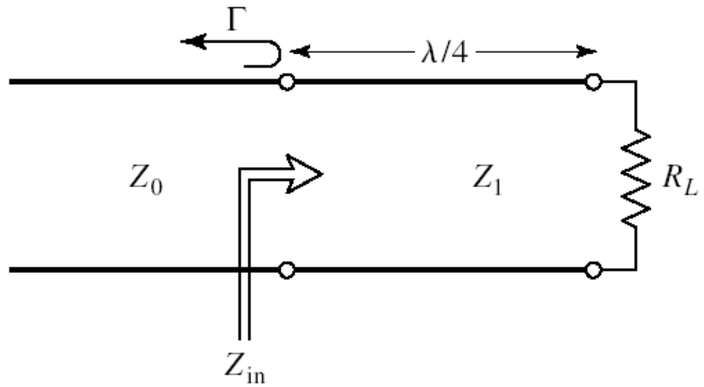


$$Z_{in} = Z_1 \frac{1+\Gamma e^{-2j\beta l}}{1-\Gamma e^{-2j\beta l}}$$

$$\Gamma_O = \frac{V_0^-}{V_0^+} = \frac{R_L - Z_1}{R_L + Z_1}$$

$$Z_{in} = Z_1 \frac{R_L + jZ_1 \tan(\beta l)}{Z_1 + jR_L \tan(\beta l)}$$

The quarter-wave transformer



$$\Gamma_{in} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0}$$

$$\beta \cdot l = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \frac{\pi}{2} \qquad Z_{in} = \frac{Z_1^2}{R_L}$$

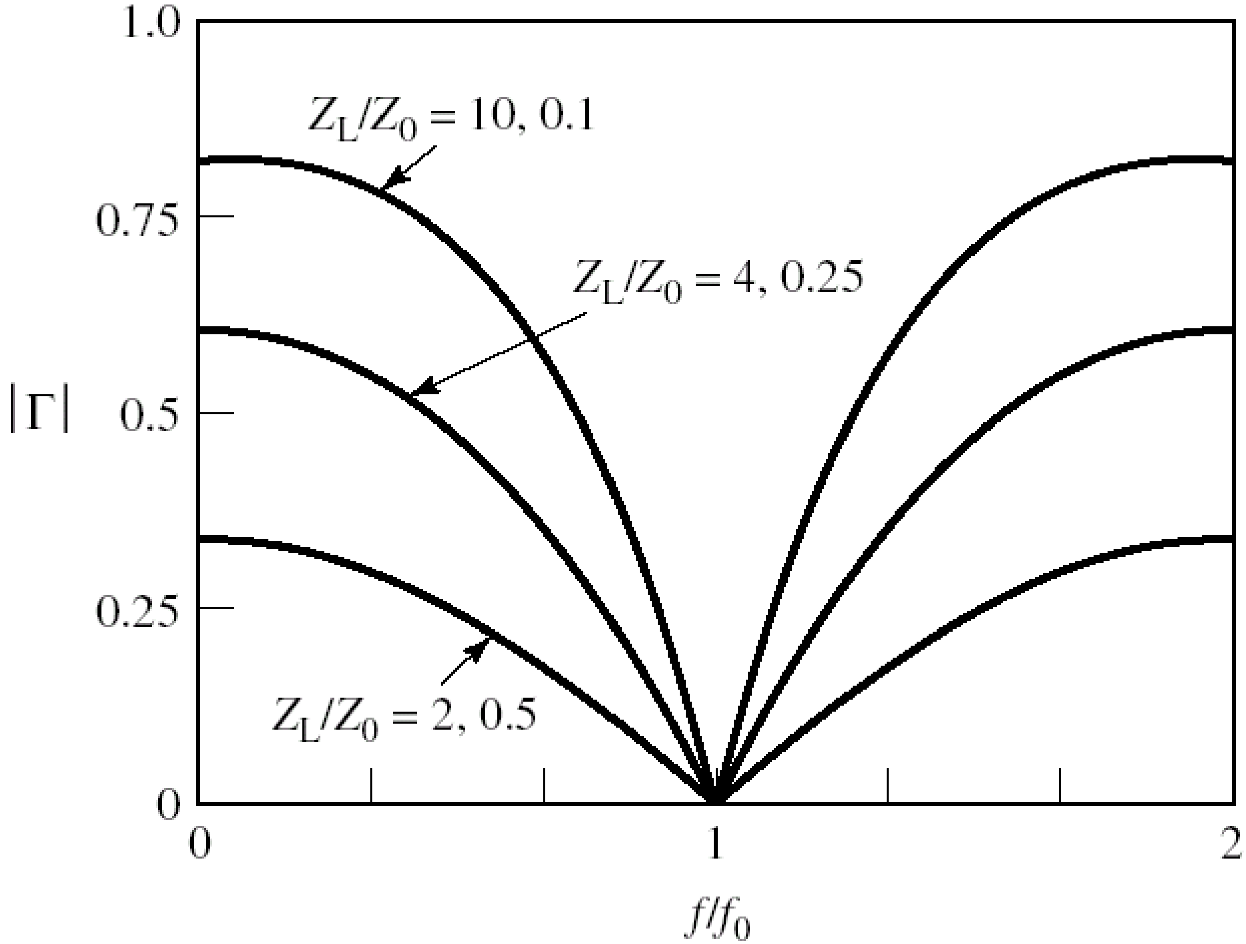
$$\Gamma_{in} = \frac{Z_1^2 - Z_0 \cdot R_L}{Z_1^2 + Z_0 \cdot R_L} \qquad \Gamma_{in} = 0 \quad Z_1 = \sqrt{Z_0 R_L}$$

- In the feed line (Z_o) we have only progressive wave
- In the quarter-wave line (Z_1) we have standing waves

Frequency response

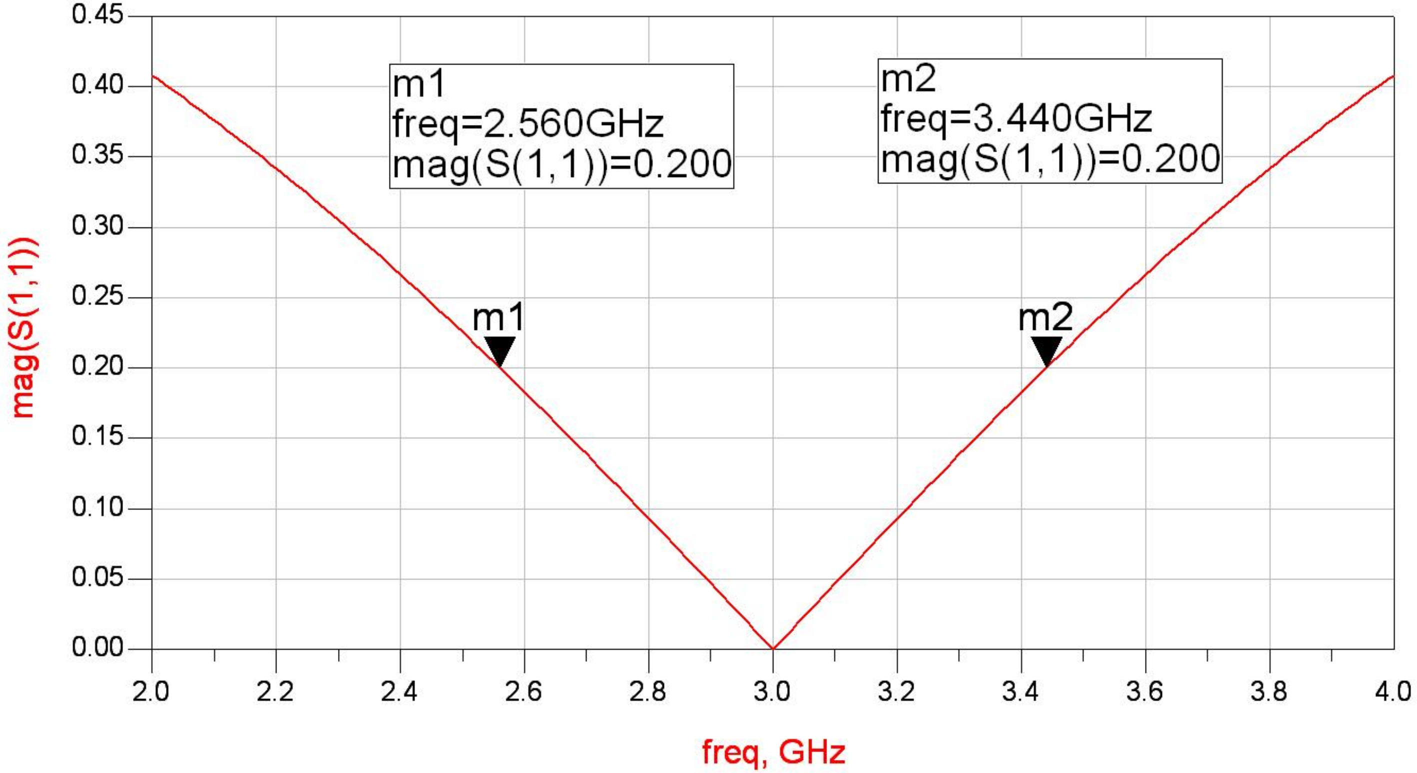
- Bandwidth depends on the initial mismatch

increased bandwidth
for smaller load
mismatches



Simulation

- ADS Simulation

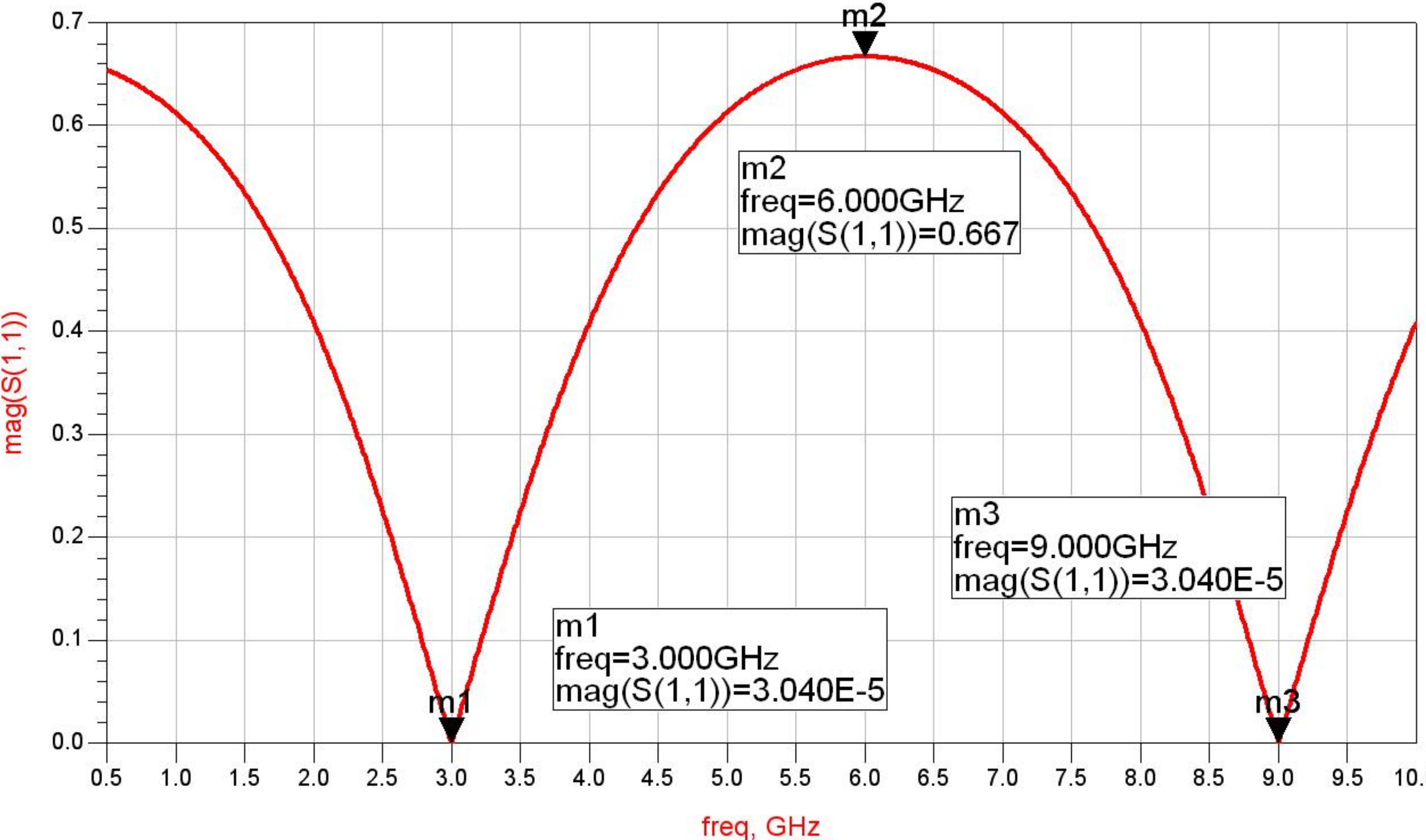


$$\Delta f = 0.88 GHz$$

$$\left|\Gamma(3GHz)\right| = 3 \cdot 10^{-5}$$

$$\frac{\Delta f}{f_0} = \frac{0.88}{3} = 0.2933$$

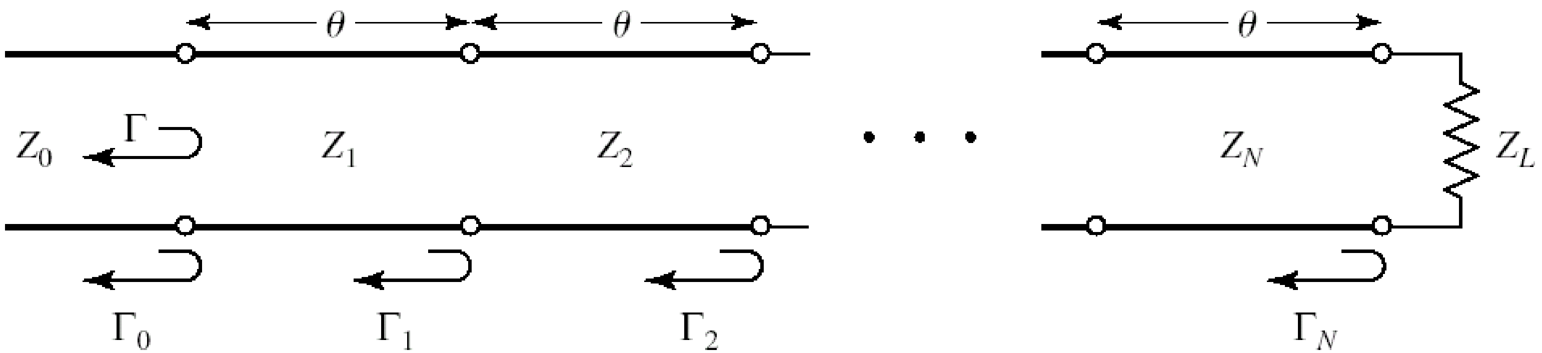
Full bandwidth simulation



Multisection Impedance Transformer

- The quarter-wave transformer can match any real load to any feed line impedance
- If a greater bandwidth for the match is required we must use multiple sections of transmission lines transformers:
  - binomial
  - Chebyshev

Multisection transformers



$$\Gamma_1 = \frac{Z_1 - Z_0}{Z_1 + Z_0}$$

$$\Gamma_n = \frac{Z_{n+1} - Z_n}{Z_{n+1} + Z_n}$$

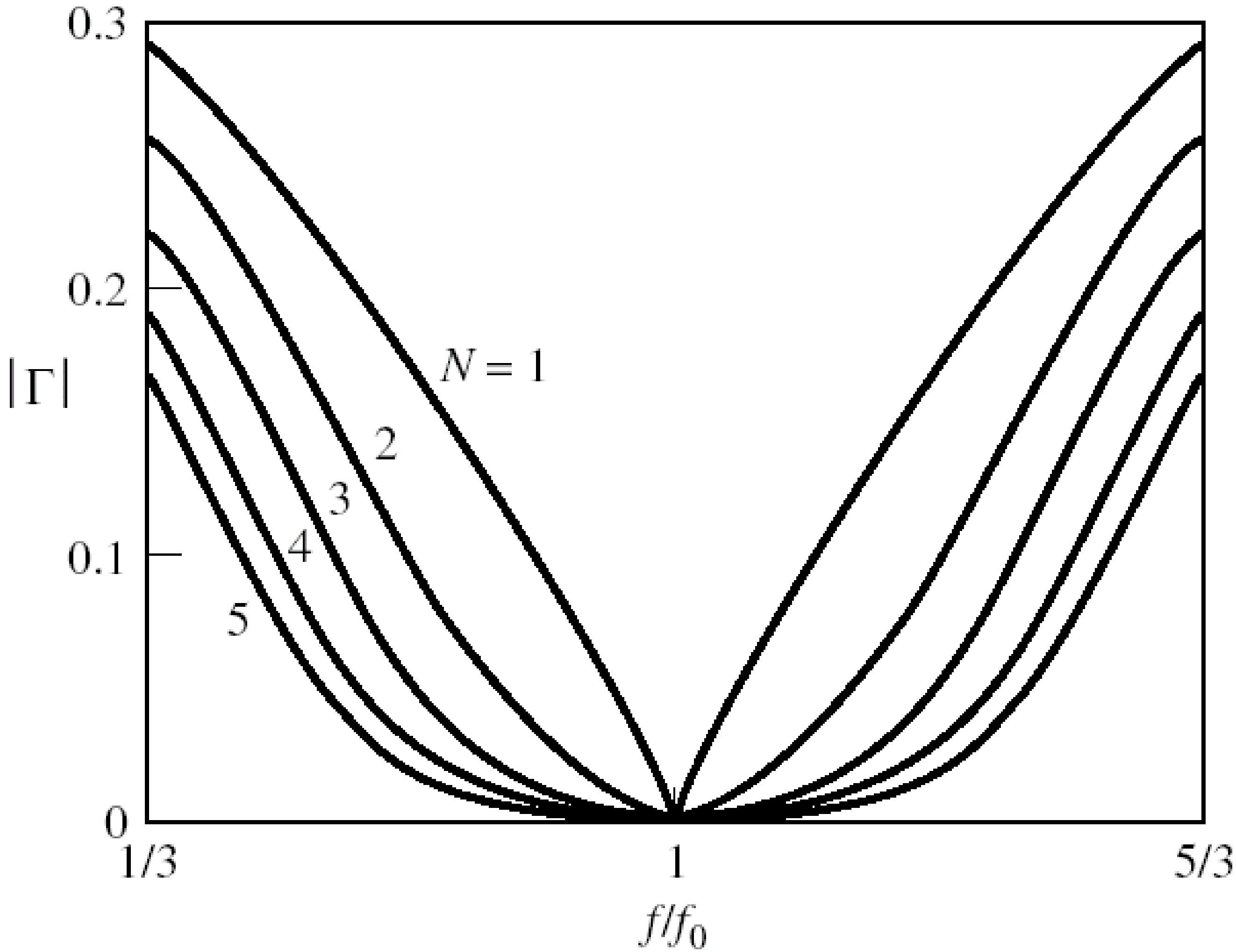
$$n = \overline{1, N-1}$$

$$\Gamma_N = \frac{Z_L - Z_N}{Z_L + Z_N}$$

- We also assume that all impedances **increase or decrease monotonically** across the transformer
- This implies that all reflection coefficients will be real and of the same sign
- Previously, 1 section $\Gamma \cong \Gamma_1 + \Gamma_3 \cdot e^{-2j\theta} \Rightarrow$

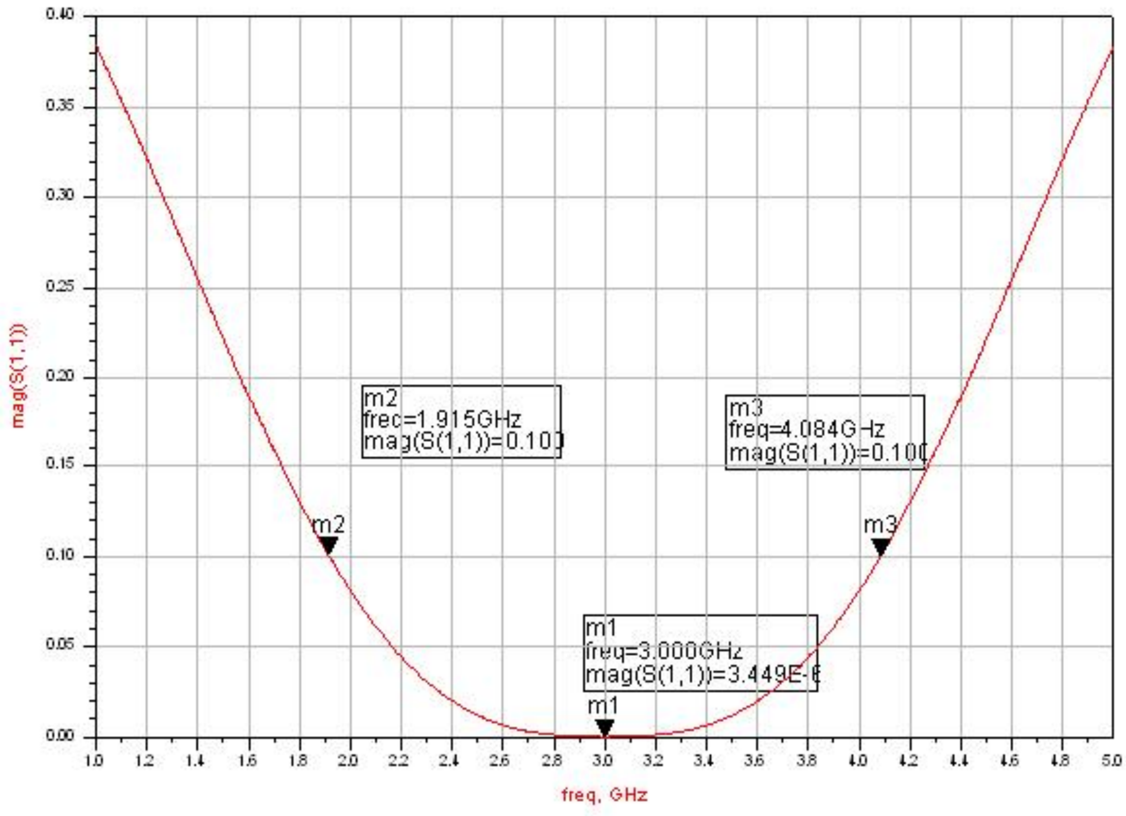
$$\Gamma(\theta) = \Gamma_0 + \Gamma_1 \cdot e^{-2j\theta} + \Gamma_2 \cdot e^{-4j\theta} + \cdots + \Gamma_N \cdot e^{-2jN\theta}$$

Bandwidth / Binomial



Simulation

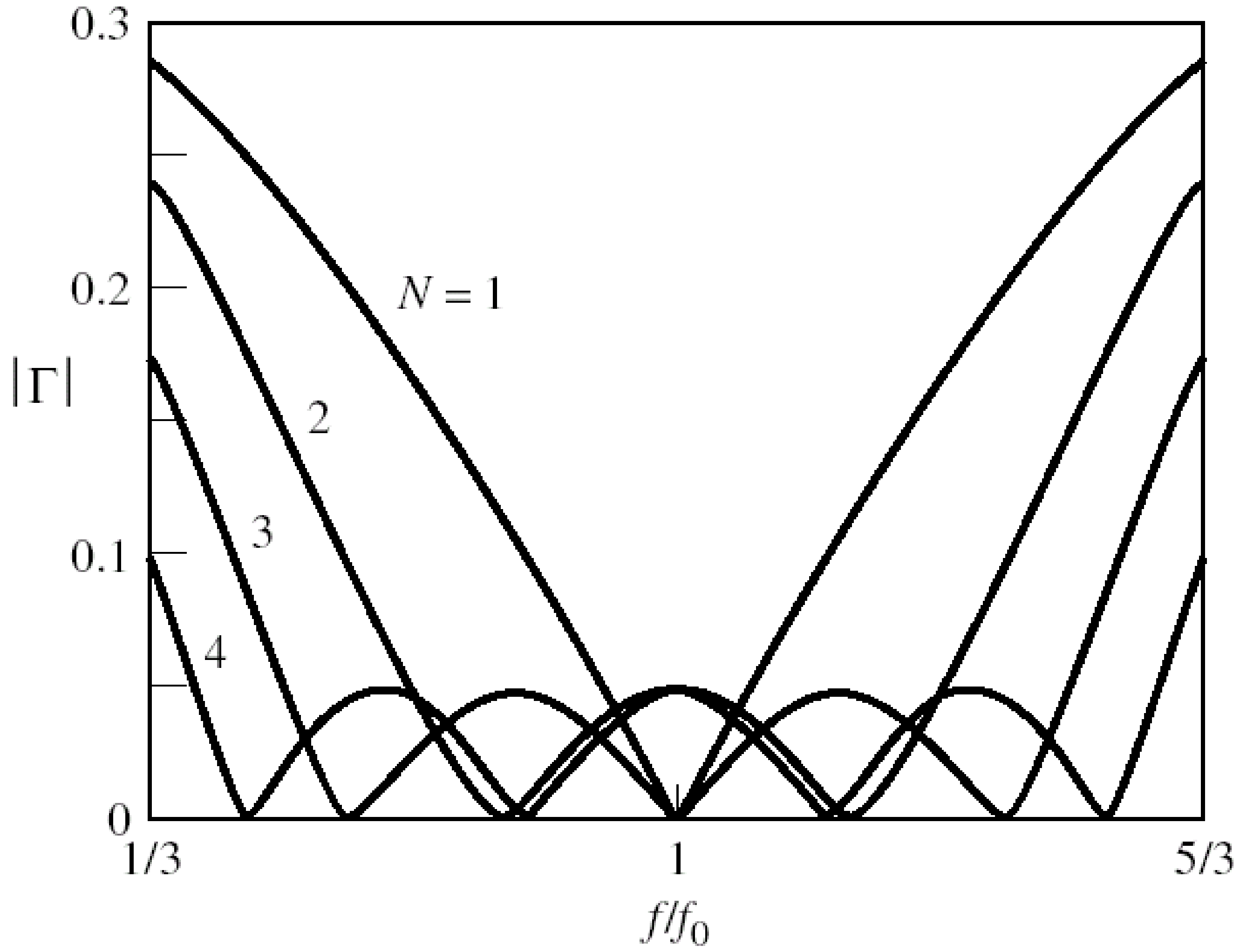
- Similarly Lab. 1



$$\Delta f = 2.169 GHz$$

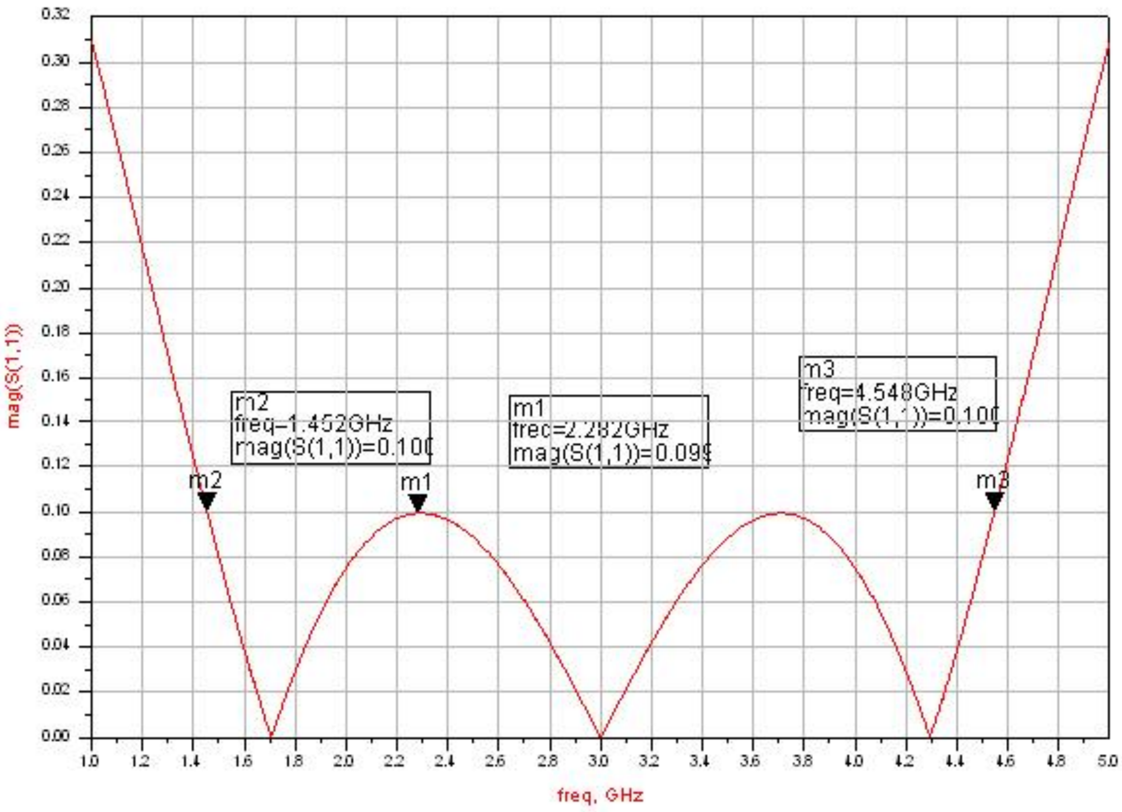
$$|\Gamma(3GHz)| = 3.5 \cdot 10^{-6}$$

Bandwidth / Chebyshev



Simulation

- Similarly Lab. 1



$$\Delta f = 3.096GHz$$

$$\left|\Gamma(3GHz)\right| = 4.17 \cdot 10^{-5}$$

$$\left|\Gamma(2.282GHz)\right| = 0.09925$$

Contact

- Microwave and Optoelectronics Laboratory
- **http://rf-opto.etti.tuiasi.ro**
- **rdamian@etti.tuiasi.ro**